Solutions

Each answer shows the steps, then the **result**.

2.1

- $\det = 5(4) - 2(3) = 14$

2.2

- $\det = 2 - 1 = 1$; inverse $= \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$

3.1 B

- A scalar multiple of I is an enlargement; here scale factor 2

3.2

(a)

- $AB = \begin{pmatrix} 1\cdot 2 + 2\cdot 1 & 1\cdot 0 + 2\cdot 3 \\ 3\cdot 2 + 4\cdot 1 & 3\cdot 0 + 4\cdot 3 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 10 & 12 \end{pmatrix}$

(b)

- $\det(AB) = 48 - 60 = -12$; $\det A = -2$, $\det B = 6$, product $= -12$ ✓

3.3

(a)

- area scale factor $= \det M = 6$

(b)

- $M^{-1} = \frac{1}{6}\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$; this matrix undoes T

3.4

(a)

- $\det \mathbf{M} = 4 - 1 = \mathbf{3}$; non-singular, since $\det \mathbf{M} \neq 0$

(b)

- invariant points satisfy $2x + y = x$ and $x + 2y = y$; both give $x + y = 0$
- the line $\mathbf{y} = -\mathbf{x}$

(c)

- $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} t \\ t \end{pmatrix} = \begin{pmatrix} 3t \\ 3t \end{pmatrix}$, which lies on $y = x$, so the line maps to itself
- but (t, t) maps to $(3t, 3t)$, a different point unless $t = 0$, so its points are not invariant

(d)

- $\mathbf{M}^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$
- $\mathbf{N} = \begin{pmatrix} -5 & -4 \\ -4 & -5 \end{pmatrix}$