A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

1.3 Summation of series

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS method of differences 差分法 • partial fractions 部分分式 • sum to infinity 无穷和 • summation of series 级数求和

Objective

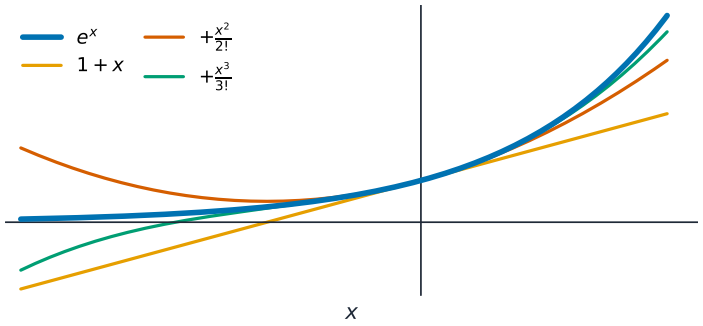
Build the skills to answer exam questions on the **summation of series** 级数求和 — the standard results for $\sum r$, $\sum r^2$, $\sum r^3$, and the **method of differences** 差分法.

You must be able to:

- use $\sum r = \frac{1}{2}n(n+1)$, $\sum r^2 = \frac{1}{6}n(n+1)(2n+1)$, $\sum r^3 = \frac{1}{4}n^2(n+1)^2$
- sum a series by the method of differences (telescoping)
- decide convergence and find a sum to infinity

1 Worked examples

■ **Standard results & differences**



$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \cdot \quad e^x = 1 + x + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} + \cdots$$

A finite sum builds from standard results; telescoping cancels all but the end terms

$$\sum_{r=1}^{n}(2r+1) = 2\cdot\tfrac{1}{2}n(n+1) + n = n(n+2).$$

Differences: if $u_r = f(r) - f(r+1)$ then $\sum_{r=1}^{n} u_r = f(1) - f(n+1)$.

2 Practice

2.1 Evaluate $\sum_{r=1}^{n} r^2$ as a formula in n. [1]

2.2 Find $\sum_{r=1}^{10}(3r-1)$. [2]

3 Exam-style questions

3.1 $\sum_{r=1}^{n} r^3$ equals: [1]

A B
C D

A $\frac{1}{4}n^2(n+1)^2$
B $\frac{1}{6}n(n+1)(2n+1)$
C $\frac{1}{2}n(n+1)$
D n^3

3.2 (a) Express $\dfrac{1}{r(r+1)}$ in partial fractions. [2]

(b) Hence find $\sum_{r=1}^{n} \dfrac{1}{r(r+1)}$ by the method of differences, and state its sum to infinity. [4]

3.3 Find $\sum_{r=1}^{n}(r^2+r)$, giving your answer as a single factorised expression. [4]