

4.4 Non-parametric tests

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS wilcoxon signed-rank 符号秩 • wilcoxon signed-rank test 威尔科克森符号秩检验
• rank-sum 秩和 • sign test 符号检验 • non-parametric tests 非参数检验

Objective

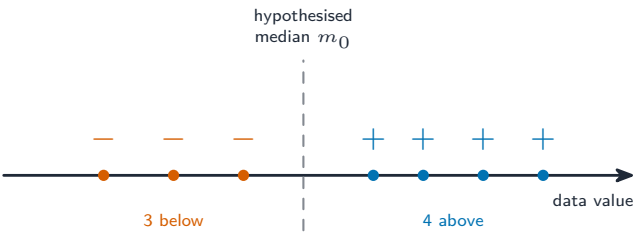
Build the skills to answer exam questions on **non-parametric tests** 非参数检验 — the **sign test** 符号检验, the **Wilcoxon signed-rank** 符号秩 (matched-pairs) test, and the **rank-sum** 秩和 test.

You must be able to:

- carry out a sign test using a binomial model
- carry out a Wilcoxon signed-rank test on paired data
- choose the right non-parametric test and state when to use one

1 Worked examples

■ The sign test



The sign test counts values above/below a proposed median, then tests the counts with a binomial model

Sign test for median $= m_0$: count + and - signs; under H_0 the number of + is $\sim B(n, 0.5)$; a very lopsided count is significant.

2 Practice

2.1 State when a non-parametric test is preferred over a t-test. [1]

2.2 In a sign test, state the distribution of the number of positive signs under H_0 . [1]

3 Exam-style questions

3.1 The Wilcoxon signed-rank test uses, in addition to the signs of the differences: [1]

- **A** nothing else
- **B** the sizes (ranks) of the differences
- **C** the sample mean
- **D** the population variance

3.2 Ten people rate a product before and after a change. Eight rate it higher (+) and two lower (-).

(a) State H_0 and H_1 for a sign test of "no change" vs "an improvement". [2]

(b) Using $X \sim B(10, 0.5)$, find $P(X \geq 8)$ and state the conclusion at the 5% level. [4]

3.3 Explain why a non-parametric test might be chosen instead of a t-test for a small sample. [2]

Solutions

2.1 when the data cannot be assumed to be normally distributed.

2.2 $\sim B(n, 0.5)$.

3.1 B. The signed-rank test also uses the ranks (sizes) of the differences.

3.2 (a) H_0 : median difference = 0 (no change); H_1 : median difference > 0 (improvement).

(b) $P(X \geq 8) = P(8) + P(9) + P(10) = \frac{45 + 10 + 1}{1024} = \frac{56}{1024} = 0.0547$; $0.0547 > 0.05$, so do not reject H_0 — insufficient evidence of improvement.

3.3 a t-test assumes the population is normal; for a small sample that assumption cannot be checked reliably; a non-parametric test makes no such assumption, so it is safer when normality is doubtful.