

4.3 Chi-squared tests

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS goodness of fit 拟合优度 • contingency table 列联表 • independence 独立性
• chi-squared test 卡方检验 • degrees of freedom 自由度

Objective

Build the skills to answer exam questions on **chi-squared tests** 卡方检验 — **goodness of fit** 拟合优度 and **independence** 独立性 in a **contingency table** 列联表.

You must be able to:

- compute expected counts and $\chi^2 = \sum \frac{(O - E)^2}{E}$
- find the degrees of freedom and compare with the critical value
- carry out a goodness-of-fit and an independence test

1 Worked examples

■ Goodness of fit

reject H_0 if $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$

χ^2 with 3 d.f.

critical value
7.815

reject region
(5%)

Reject the model when χ^2 exceeds the critical value (in the shaded tail)

Four equal categories, observed 20,30,25,25 (each $E = 25$): $\chi^2 = \frac{25 + 25 + 0 + 0}{25} = 2$. With 3 d.f. the 5% value is 7.815; $2 < 7.815 \rightarrow$ do not re-

ject.

2 Practice

2.1 Write down the χ^2 test statistic formula. [1]

2.2 A goodness-of-fit test has 5 categories and no estimated parameters. State the degrees of freedom. [1]

3 Exam-style questions

3.1 In a χ^2 test, a **large** value of χ^2 suggests: [1]

- **A** observed and expected agree well
- **B** observed and expected differ a lot
- **C** the sample is small
- **D** the test is invalid

3.2 A die is rolled 120 times, giving the counts:

Face	1	2	3	4	5	6
Observed	15	22	23	19	18	23

Test at the 5% level whether the die is fair ($\chi^2_{\text{crit}} = 11.07$, 5 d.f.). [6]

3.3 For a 3×4 contingency table, state the number of degrees of freedom for a test of independence. [2]

Solutions

2.1 $\chi^2 = \sum \frac{(O - E)^2}{E}$.

2.2 $5 - 1 = 4$.

3.1 B. A large χ^2 means observed counts differ greatly from expected.

3.2 each $E = \frac{120}{6} = 20$; $\chi^2 = \frac{(15 - 20)^2 + (22 - 20)^2 + (23 - 20)^2 + (19 - 20)^2 + (18 - 20)^2 + (23 - 20)^2}{20}$
 $= \frac{25 + 4 + 9 + 1 + 4 + 9}{20} = \frac{52}{20} = 2.6$; $2.6 < 11.07$; do not reject — no evidence the die is unfair.

3.3 degrees of freedom $= (3 - 1)(4 - 1) = 6$.