A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

4.3 Chi-squared tests

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

Objective

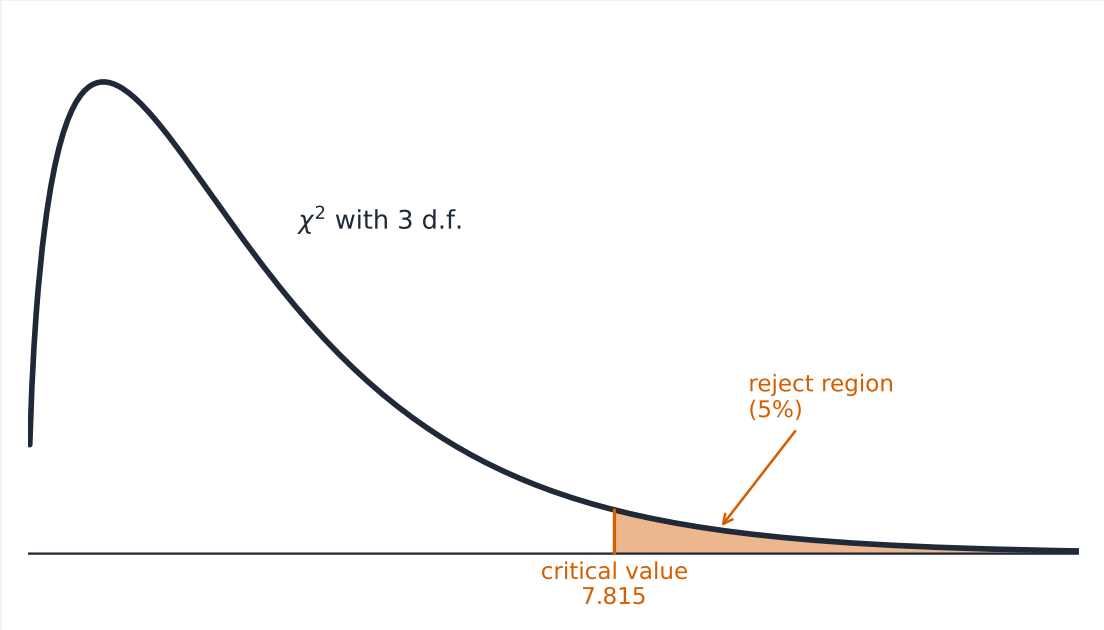
Build the skills to answer exam questions on **chi-squared tests** 卡方检验—**goodness of fit** 拟合优度 and **independence** 独立性 in a **contingency table** 列联表.

You must be able to:

- compute expected counts and $\chi^2 = \sum \dfrac{(O-E)^2}{E}$
- find the degrees of freedom and compare with the critical value
- carry out a goodness-of-fit and an independence test

1 Worked examples

■ Goodness of fit



Reject the model when χ^2 exceeds the critical value (in the shaded tail)

Four equal categories, observed $20, 30, 25, 25$ **(each** $E = 25$**):** $\chi^2 = \dfrac{25+25+0+0}{25} = 2$. With 3 d.f. the 5% value is 7.815; $2 < 7.815 \rightarrow$ do not reject.

2 Practice

2.1 Write down the χ^2 test statistic formula. [1]

2.2 A goodness-of-fit test has 5 categories and no estimated parameters. State the degrees of freedom. [1]

3 Exam-style questions

3.1 In a χ^2 test, a **large** value of χ^2 suggests: [1]

- **A** observed and expected agree well
- **B** observed and expected differ a lot
- **C** the sample is small
- **D** the test is invalid

3.2 A die is rolled 120 times, giving the counts:

Face	**1**	**2**	**3**	**4**	**5**	**6**
Observed	15	22	23	19	18	23

Test at the 5% level whether the die is fair ($\chi^2_{\text{crit}} = 11.07$, 5 d.f.). [6]

3.3 For a 3×4 contingency table, state the number of degrees of freedom for a test of independence. [2]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **4.3 Chi-squared tests** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/41 N25 —Q2 (goodness of fit)
- 9231/42 N25 —Q3 (chi-squared)
- 9231/44 N25 —Q5 (independence test)

Solutions

2.1 $\chi^2 = \sum \dfrac{(O-E)^2}{E}$.

2.2 $5 - 1 = 4$.

3.1 **B**. A large χ^2 means observed counts differ greatly from expected.

3.2 each $E = \dfrac{120}{6} = 20$; $\chi^2 = \dfrac{(15-20)^2 + (22-20)^2 + (23-20)^2 + (19-20)^2 + (18-20)^2 + (23-20}{20}$ $= \dfrac{25+4+9+1+4+9}{20} = \dfrac{52}{20} = 2.6$; $2.6 < 11.07$; do not reject —no evidence the die is unfair.

3.3 degrees of freedom $= (3-1)(4-1) = 6$.