A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

3.6 Momentum

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

Objective

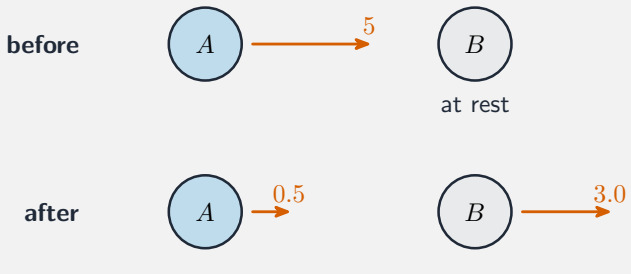
Build the skills to answer exam questions on **momentum and impacts** 动量与碰撞—conservation of momentum, the **coefficient of restitution** 恢复系数 e, and **oblique impact** 斜碰撞. Take $g = 10$ m s^{-2}.

You must be able to:

- apply conservation of momentum to a collision
- use $e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$
- resolve velocities for an oblique impact

1 Worked examples

Restitution



$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{3.0 - 0.5}{5} = 0.5$$

e compares how fast the bodies separate with how fast they approached ($0 \leqslant e \leqslant 1$)

A (2 kg, 5 m s^{-1}) hits stationary B (3 kg), $e = 0.5$: momentum $2(5) = 2v_A + 3v_B$; restitution $v_B - v_A = 0.5(5) = 2.5$. Solve: $v_A = 0.5$, $v_B = 3.0$ m s^{-1}.

2 Practice

2.1 State the range of possible values of the coefficient of restitution e. [1]

2.2 Two balls approach at a combined closing speed of 8 m s^{-1} with $e = 0.25$. Find their separation speed. [2]

3 Exam-style questions

3.1 A perfectly elastic collision has a coefficient of restitution: [1]

- **A** 0
- **B** 0.5
- **C** 1
- **D** 2

3.2 A sphere P of mass 3 kg moving at 4 m s^{-1} collides directly with a sphere Q of mass 1 kg moving towards it at 2 m s^{-1}. The coefficient of restitution is 0.5.

(a) Write the momentum and restitution equations. [3]

(b) Find the velocities of P and Q after impact. [4]

3.3 A ball is dropped from a height of 2 m onto the floor, with coefficient of restitution 0.8 between ball and floor. Find the height of the first bounce. [4]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **3.6 Momentum** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/31 N25 —Q1 (momentum / restitution)
- 9231/32 N25 —Q5 (impact)
- 9231/34 N25 —Q7 (collision)

Solutions

2.1 $0 \leqslant e \leqslant 1$.

2.2 separation $= e\times$ approach $= 0.25 \times 8 = 2$ m s^{-1}.

3.1 C. A perfectly elastic collision has $e = 1$.

3.2 (a) take P's direction positive: momentum $3(4)+1(-2) = 3v_P+1v_Q$, i.e. $3v_P+v_Q = 10$; restitution $v_Q - v_P = 0.5(4-(-2)) = 3$.
(b) from the two equations: $v_Q = v_P+3$; $3v_P+v_P+3 = 10 \Rightarrow 4v_P = 7 \Rightarrow v_P = 1.75$ m s^{-1}; $v_Q = 4.75$ m s^{-1}.

3.3 speed hitting floor: $v = \sqrt{2g(2)} = \sqrt{40} = 6.32$ m s^{-1}; rebound speed $= ev = 0.8(6.32) = 5.06$ m s^{-1}; bounce height $= \dfrac{(ev)^2}{2g} = \dfrac{25.6}{20} = 1.28$ m.