3.5 Linear motion under a variable force

Name: ______________ Class: __________ Date: ____________ **Total: 14 marks**

Objective

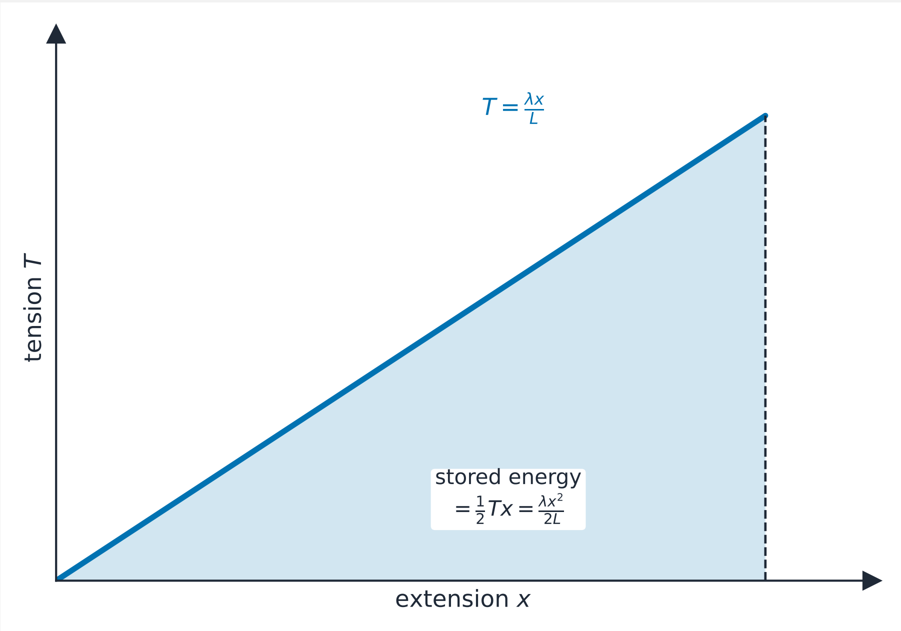
Build the skills to answer exam questions on **linear motion under a variable force 变力下的直线运动**—using $a = v\dfrac{dv}{dx}$ (force depends on position) or $a = \dfrac{dv}{dt}$ (force depends on time). Take $g = 10$ m s^{-2}.

You must be able to:

- choose $a = v\dfrac{dv}{dx}$ for a position-dependent force
- set up and solve the resulting differential equation
- use $a = \dfrac{dv}{dt}$ for a time-dependent force

1 Worked examples

■ Force depending on position



A position-dependent force (e.g. a spring) makes Newton's law a differential equation in v and x

8 **kg under force** $(x^3 + 4x)$ **N,** $v = 1$ **at** $x = 0$**:** $8v\dfrac{dv}{dx} = x^3 + 4x$; integrate →

$4v^2 = \frac{1}{4}x^4 + 2x^2 + 4$, so $v = \frac{1}{4}x^2 + 1$.

2 Practice

2.1 State the form of acceleration to use when a force depends on **position** x. [1]

2.2 A particle has $v\dfrac{dv}{dx} = 2x$. Find v^2 in terms of x, given $v = 0$ at $x = 0$. [2]

3 Exam-style questions

3.1 For a force that depends on position, acceleration is best written as: [1]

- **A** $\dfrac{dv}{dt}$
- **B** $v\dfrac{dv}{dx}$
- **C** $\dfrac{dx}{dt}$
- **D** $\dfrac{d^2x}{dt^2}$ only

3.2 A particle of mass 2 kg moves along a straight line under a force $F = 6x$ N in the direction of motion. At $x = 0$ the speed is 2 m s^{-1}.

(a) Form a differential equation in v and x. [2]

(b) Find v when $x = 2$. [4]

3.3 A particle of mass 0.5 kg moves so that its acceleration is $\dfrac{dv}{dt} = 4 - 2t$. At $t = 0$ it is at rest. Find its velocity when $t = 3$ and comment. [4]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **3.5 Linear motion under a variable force** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/31 N25 —Q3 (variable force)
- 9231/32 N25 —Q7 (variable force)
- 9231/34 N25 —Q5 (motion under a force)

Solutions

2.1 $a = v\dfrac{dv}{dx}$.

2.2 $\int v\,dv = \int 2x\,dx \Rightarrow \frac{1}{2}v^2 = x^2 + C$; $v = 0$ at $x = 0 \Rightarrow C = 0$, so $v^2 = 2x^2$.

3.1 **B**. Use $a = v\dfrac{dv}{dx}$ for a position-dependent force.

3.2 (a) $2v\dfrac{dv}{dx} = 6x$, i.e. $v\dfrac{dv}{dx} = 3x$.
(b) $\int v\,dv = \int 3x\,dx \Rightarrow \frac{1}{2}v^2 = \frac{3}{2}x^2 + C$; at $x = 0$, $v = 2 \Rightarrow C = 2$; at $x = 2$: $\frac{1}{2}v^2 = 6 + 2 = 8 \Rightarrow v^2 = 16$; $v = 4$ m s^{-1}.

3.3 $v = \int(4-2t)\,dt = 4t-t^2+C$; $v = 0$ at $t = 0 \Rightarrow C = 0$; at $t = 3$: $v = 12-9 = 3$ m s^{-1}; the force has reversed (since $4 - 2t < 0$ for $t > 2$), so the particle is slowing after $t = 2$.