A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

3.4 Hooke's law

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS elastic potential energy 弹性势能 • hooke's law 胡克定律 • equilibrium 平衡

Objective

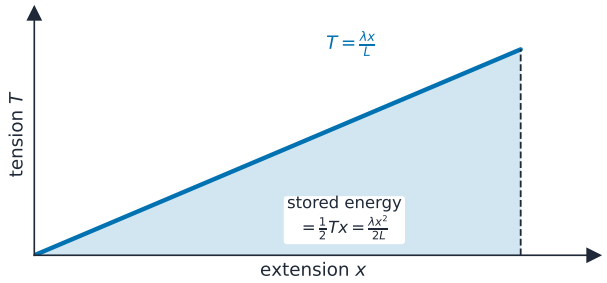
Build the skills to answer exam questions on **Hooke's law** 胡克定律 — the tension $T = \dfrac{\lambda x}{L}$ in an elastic string/spring, and the stored **elastic potential energy** 弹性势能 $E = \dfrac{\lambda x^2}{2L}$. Take $g = 10$ m s^{-2}.

You must be able to:

- use $T = \dfrac{\lambda x}{L}$ to find tension, extension, or λ
- find the stored energy $E = \dfrac{\lambda x^2}{2L}$
- combine Hooke's law with equilibrium or energy conservation

1 Worked examples

Tension and energy



$T = \frac{\lambda}{L}x$ · $E = \frac{1}{2}Tx = \frac{\lambda x^2}{2L}$ (area under graph)

Tension rises in proportion to extension; the shaded triangle is the stored elastic energy

Natural length 2 **m, modulus** 50 **N, extension** 0.5 **m:** $T = \dfrac{50 \times 0.5}{2} = 12.5$ N;

$E = \dfrac{50 \times 0.5^2}{2 \times 2} = 3.125$ J.

2 Practice

2.1 A spring of natural length 1 m and modulus 30 N is stretched by 0.2 m. Find the tension. [2]

2.2 Find the energy stored in the spring in 2.1. [2]

3 Exam-style questions

3.1 Doubling the extension of an elastic string multiplies the stored energy by: [1]

- **A** 2
- **B** 4
- **C** $\frac{1}{2}$
- **D** 1

3.2 A particle of mass 2 kg hangs in equilibrium on an elastic string of natural length 1.5 m and modulus 60 N.

(a) Find the extension of the string. [3]

(b) Find the elastic energy stored. [2]

3.3 A particle of mass 0.5 kg is attached to one end of an elastic string of natural length 2 m and modulus 40 N, the other end fixed at O. The particle is released from rest at O and falls vertically. Using energy, find its speed when the string has extension 0.5 m. [5]

Solutions

2.1 $T = \dfrac{30 \times 0.2}{1} = 6$ N.

2.2 $E = \dfrac{30 \times 0.2^2}{2 \times 1} = 0.6$ J.

3.1 B. $E \propto x^2$, so doubling x multiplies E by 4.

3.2 (a) equilibrium $T = mg = 20$ N; $\dfrac{60x}{1.5} = 20 \Rightarrow 40x = 20 \Rightarrow x = 0.5$ m.

(b) $E = \dfrac{60 \times 0.5^2}{2 \times 1.5} = \dfrac{15}{3} = 5$ J.

3.3 the particle falls $2 + 0.5 = 2.5$ m to reach extension 0.5 m; energy: $mg(2.5) = \frac{1}{2}mv^2 + \dfrac{\lambda x^2}{2L}$; $0.5(10)(2.5) = \frac{1}{2}(0.5)v^2 + \dfrac{40(0.25)}{2(2)}$; $12.5 = 0.25v^2 + 2.5 \Rightarrow v^2 = 40$; $v = 6.32$ m s^{-1}.