A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

3.3 Circular motion

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

Objective

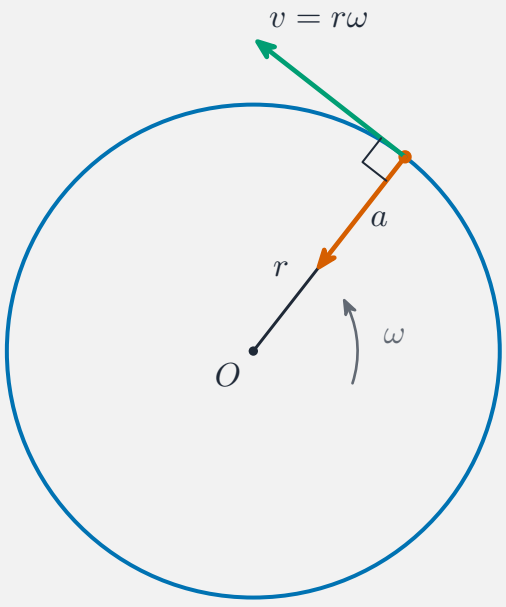
Build the skills to answer exam questions on **circular motion** 圆周运动—**angular speed** 角速度, **centripetal acceleration** 向心加速度, the **conical pendulum** 圆锥摆, and motion in a **vertical circle** 竖直圆. Take $g = 10$ m s^{-2}.

You must be able to:

- use $v = r\omega$ and $a = r\omega^2 = \dfrac{v^2}{r}$
- analyse a horizontal circle (conical pendulum)
- use energy + the centripetal equation for a vertical circle

1 Worked examples

■ Centripetal force



Velocity points along the tangent; acceleration points inward to the centre

$r = 2$ **m,** $\omega = 3$ **rad s**$^{-1}$**:** $v = r\omega = 6$ m s^{-1}; $a = r\omega^2 = 18$ m s^{-2}.

Centripetal force $= ma = \dfrac{mv^2}{r}$, provided by tension, friction or a component of

> weight.

2 Practice

2.1 A particle moves in a circle of radius 0.5 m at 4 m s^{-1}. Find its acceleration. [2]

2.2 Convert an angular speed of 120 rev/min to rad s^{-1}. [2]

3 Exam-style questions

3.1 The centripetal acceleration of a particle moving at speed v in a circle of radius r is: [1]

- **A** $\frac{v}{r}$
- **B** $\frac{v^2}{r}$
- **C** vr
- **D** $\frac{r}{v^2}$

3.2 A particle of mass 0.2 kg is attached to a string of length 0.5 m and whirled in a horizontal circle on a smooth table at 3 rev s^{-1}.

(a) Find the angular speed in rad s^{-1}. [2]

(b) Find the tension in the string. [3]

3.3 A conical pendulum has a bob of mass 0.4 kg on a string of length 1 m, which makes

an angle of 30° with the vertical.

(a) Find the tension in the string. [3]

(b) Find the angular speed of the bob. [3]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **3.3 Circular motion** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/31 N25 —Q2 (circular motion)
- 9231/31 N25 —Q6 (vertical circle)
- 9231/32 N25 —Q1 (conical pendulum)

Solutions

2.1 $a = \dfrac{v^2}{r} = \dfrac{16}{0.5} = 32$ m s^{-2}.

2.2 120 rev/min = 2 rev/s = $2 \times 2\pi = 4\pi = 12.6$ rad s^{-1}.

3.1 B. $a = \dfrac{v^2}{r}$ (also $= r\omega^2$).

3.2 (a) $\omega = 3 \times 2\pi = 6\pi = 18.8$ rad s^{-1}.
(b) $T = mr\omega^2 = 0.2 \times 0.5 \times (6\pi)^2 = 0.2 \times 0.5 \times 355.3 = 35.5$ N.

3.3 (a) vertical: $T\cos 30^\circ = mg = 4 \Rightarrow T = \dfrac{4}{0.866} = 4.62$ N.
(b) horizontal: $T\sin 30^\circ = mr\omega^2$ with $r = 1 \times \sin 30^\circ = 0.5$; $4.62 \times 0.5 = 0.4 \times 0.5 \times \omega^2 \Rightarrow \omega^2 = 11.55 \Rightarrow \omega = 3.40$ rad s^{-1}.