A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

# 3.2 Equilibrium of a rigid body

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

**KEY WORDS** centre of mass 质心 • equilibrium 平衡 • toppling 翻倒 • sliding 滑动 • moment 力矩

## Objective

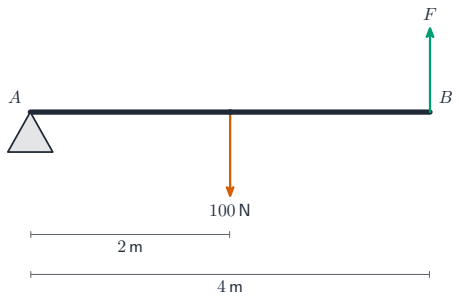
Build the skills to answer exam questions on the **equilibrium of a rigid body** 刚体平衡 — **moments** 力矩, the **centre of mass** 质心, and conditions for **toppling** 翻倒 or **sliding** 滑动. Take $g = 10\ \text{m s}^{-2}$.

**You must be able to:**

- take moments about a point (force × perpendicular distance)
- apply the two equilibrium conditions (forces and moments both balance)
- decide whether a body topples or slides first

## 1 Worked examples

### Taking moments




*Taking moments about the pivot $A$ balances the turning effects*

**Uniform beam $AB$, length 4 m, weight 100 N, pivot at $A$, force $F$ at $B$:** moments about $A \rightarrow F \times 4 = 100 \times 2 \Rightarrow F = 50$ N.



## 2 Practice

**2.1** A force of 20 N acts 3 m from a pivot, at right angles. Find its moment. [2]

**2.2** State the two conditions for a rigid body to be in equilibrium. [2]

## 3 Exam-style questions

**3.1** The moment of a force about a point is force multiplied by: [1]

- **A** time
- **B** the perpendicular distance
- **C** mass
- **D** velocity

**3.2** A uniform ladder $AB$ of weight 200 N and length 6 m rests with $A$ on rough ground and $B$ against a smooth vertical wall, at 60° to the horizontal.

(a) By taking moments about $A$, find the reaction at the wall. [4]

(b) Hence find the friction force at the ground. [2]

**3.3** A uniform rod $AB$ of length 2 m and weight 40 N rests horizontally on supports at $A$ and at $C$, where $C$ is 1.5 m from $A$. Find the reaction at each support. [4]



## Solutions

**2.1** moment $= 20 \times 3 = 60$ N m.

**2.2** the resultant force is zero; the resultant moment about any point is zero.

**3.1 B**. Moment = force × perpendicular distance.

**3.2** (a) wall reaction $S$ is horizontal at $B$; weight 200 N acts at the midpoint (3 m along). Moments about $A$: $S \times 6\sin 60° = 200 \times 3\cos 60°$; $S \times 5.196 = 300 \Rightarrow S = 57.7$ N.
(b) horizontal equilibrium: friction $= S = 57.7$ N.

**3.3** moments about $A$: $R_C \times 1.5 = 40 \times 1 \Rightarrow R_C = 26.7$ N; vertical: $R_A + R_C = 40 \Rightarrow R_A = 13.3$ N.