1.6 Vectors

Name: ______________ Class: __________ Date: ____________ **Total: 17 marks**

Objective

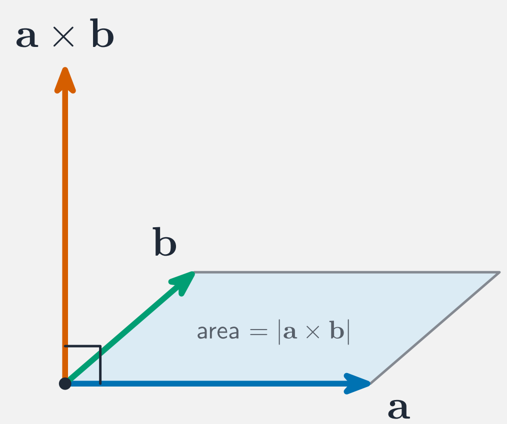
Build the skills to answer exam questions on **vectors** 向量 in 3-D —the **vector product** 向量积, equations of a **plane** 平面, and distances/angles between lines and planes.

You must be able to:

- compute $\mathbf{a} \times \mathbf{b}$ and use it for a normal/area
- write a plane as $\mathbf{r} \cdot \mathbf{n} = p$ or $ax + by + cz = d$
- find the angle between, and distance from, lines and planes

1 Worked examples

■ Vector product & plane



$\mathbf{a} \times \mathbf{b}$ is perpendicular to both; its length is the area of the parallelogram

$\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} + \mathbf{k}$: $\mathbf{a} \times \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

Plane through A with normal $\mathbf{n}$: $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$.

2 Practice

2.1 Find $\mathbf{a} \times \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{k}$. [3]

2.2 Write the plane through $(1, 2, 3)$ with normal $\mathbf{n} = (1, 1, 1)$ in the form $ax + by + cz = d$. [2]

3 Exam-style questions

3.1 $\mathbf{a} \times \mathbf{b}$ is a vector that is: [1]

- **A** parallel to $\mathbf{a}$
- **B** perpendicular to both $\mathbf{a}$ and $\mathbf{b}$
- **C** equal to $\mathbf{a} \cdot \mathbf{b}$
- **D** of length $|\mathbf{a}||\mathbf{b}| \cos \theta$

3.2 Points $A(1, 0, 2)$, $B(2, 1, 0)$ and $C(0, 2, 1)$ are given.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. [4]

(b) Hence find the equation of the plane through A, B, C. [3]

3.3 Find the acute angle between the planes $x + 2y + 2z = 5$ and $2x - y + 2z = 1$. [4]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **1.6 Vectors** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/11 N25 —Q5 (vector product / plane)
- 9231/12 N25 —Q6 (vectors)
- 9231/14 N25 —Q4 (planes / lines)

Solutions

2.1 $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \mathbf{i}(1) - \mathbf{j}(2) + \mathbf{k}(-1) = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$.

2.2 $x + y + z = 1 + 2 + 3 = 6$.

3.1 B. The vector product is perpendicular to both vectors.

3.2 (a) $\overrightarrow{AB} = (1, 1, -2)$, $\overrightarrow{AC} = (-1, 2, -1)$; cross $= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -2 \\ -1 & 2 & -1 \end{vmatrix} = \mathbf{i}(-1 + 4) - \mathbf{j}(-1 - 2) + \mathbf{k}(2 + 1) = (3, 3, 3)$.
(b) normal $(1, 1, 1)$ through $A(1, 0, 2)$: $x + y + z = 3$.

3.3 normals $(1, 2, 2)$ and $(2, -1, 2)$; dot $= 2 - 2 + 4 = 4$; $|\mathbf{n}_1| = 3$, $|\mathbf{n}_2| = 3$; $\cos\theta = \frac{4}{9} \Rightarrow \theta = 63.6°$.