A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

1.5 Polar coordinates

Name: ____________ Class: ________ Date: __________ **Total: 16 marks**

KEY WORDS polar coordinates 极坐标 • polar curves 极坐标曲线 • cartesian coordinates 直角坐标

Objective

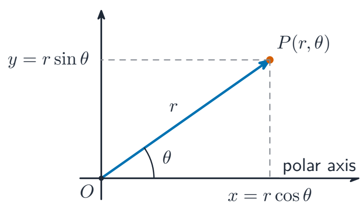
Build the skills to answer exam questions on **polar coordinates 极坐标** — converting to/from Cartesian, sketching **polar curves 极坐标曲线**, and the **sector area** $\frac{1}{2}\int r^2\,d\theta$.

You must be able to:

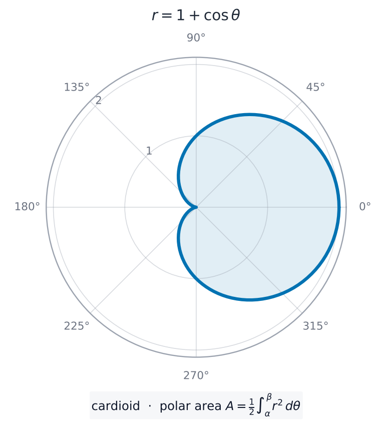
- convert between (r, θ) and (x, y)
- sketch a simple polar curve (e.g. a cardioid)
- find an area with $\frac{1}{2}\int r^2\,d\theta$

1 Worked examples

Conversion & area



$$x = r\cos\theta,\ y = r\sin\theta,\ r^2 = x^2 + y^2$$



Polar curves are traced as the angle varies

Convert $r = 4\cos\theta$**:** $\times r \to r^2 = 4r\cos\theta$, so $x^2 + y^2 = 4x$, i.e. $(x-2)^2 + y^2 = 4$ (circle).

Area: $A = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta$.

2 Practice

2.1 Convert the point $(r, \theta) = \left(2, \frac{\pi}{3}\right)$ to Cartesian coordinates. [2]

2.2 Write down the formula for the area of a polar sector. [1]

3 Exam-style questions

3.1 The polar equation $r = 2\sin\theta$ represents: [1]

- **A** a line
- **B** a circle
- **C** a parabola
- **D** a spiral

3.2 A curve has polar equation $r = 3 + 2\cos\theta$ for $0 \le \theta < 2\pi$ (a convex limaçon).

(a) Write down the greatest and least values of r, and state the value of θ at which each occurs. Explain why the curve never passes through the pole. [3]

(b) Show that the area enclosed by the curve is 11π. [5]

3.3 Convert $r = \dfrac{2}{1+\cos\theta}$ to Cartesian form. [4]

Solutions

2.1 $x = 2\cos\frac{\pi}{3} = 1$, $y = 2\sin\frac{\pi}{3} = \sqrt{3}$.

2.2 $A = \frac{1}{2}\int r^2\,d\theta$.

3.1 **B**. $r = 2\sin\theta \Rightarrow r^2 = 2r\sin\theta \Rightarrow x^2 + y^2 = 2y$, a circle.

3.2 (a) $\cos\theta$ runs from -1 to 1, so $r_{\max} = 5$ at $\theta = 0$ and $r_{\min} = 1$ at $\theta = \pi$; the least value is $1 > 0$, so r is never 0 and the curve misses the pole.
(b) $(3+2\cos\theta)^2 = 9 + 12\cos\theta + 4\cos^2\theta$, and $4\cos^2\theta = 2 + 2\cos 2\theta$, so the integrand is $11 + 12\cos\theta + 2\cos 2\theta$; $A = \frac{1}{2}\left[11\theta + 12\sin\theta + \sin 2\theta\right]_0^{2\pi}$; both sine terms vanish at 0 and 2π; $A = \frac{1}{2}(22\pi) = 11\pi$.

3.3 $r + r\cos\theta = 2 \Rightarrow r = 2 - x$ (since $r\cos\theta = x$); $r^2 = (2-x)^2 \Rightarrow x^2 + y^2 = 4 - 4x + x^2$; $y^2 = 4 - 4x$, i.e. $y^2 = -4(x-1)$.