A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

1.5 Polar coordinates

Name: ______________ Class: _________ Date: ____________ **Total: 15 marks**

Objective

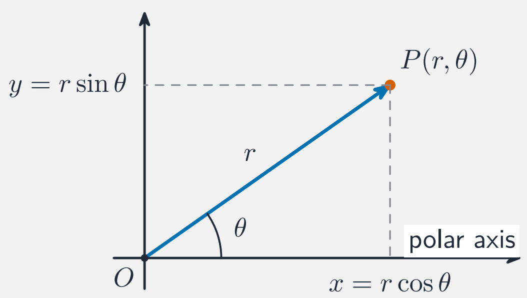
Build the skills to answer exam questions on **polar coordinates** 极坐标—converting to/from Cartesian, sketching **polar curves** 极坐标曲线, and the **sector area** $\frac{1}{2}\int r^2\,d\theta$.

You must be able to:

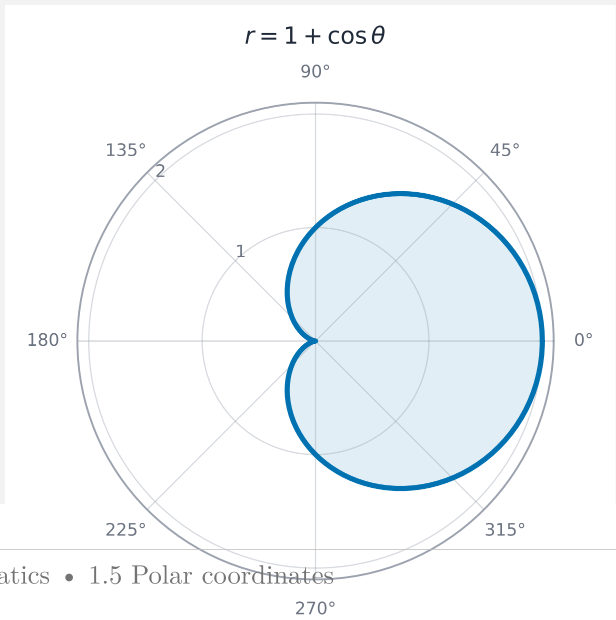
- convert between (r, θ) and (x, y)
- sketch a simple polar curve (e.g. a cardioid)
- find an area with $\frac{1}{2}\int r^2\,d\theta$

1 Worked examples

Conversion & area



$$x = r\cos\theta,\ y = r\sin\theta,\ r^2 = x^2 + y^2$$



Polar curves are traced as the angle varies

Convert $r = 4\cos\theta$**:** $\times r \to r^2 = 4r\cos\theta$, so $x^2 + y^2 = 4x$, i.e. $(x-2)^2 + y^2 = 4$ (circle).

Area: $A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta$.

2 Practice

2.1 Convert the point $(r, \theta) = \left(2, \frac{\pi}{3}\right)$ to Cartesian coordinates. [2]

2.2 Write down the formula for the area of a polar sector. [1]

3 Exam-style questions

3.1 The polar equation $r = 2\sin\theta$ represents: [1]

- **A** a line
- **B** a circle
- **C** a parabola
- **D** a spiral

3.2 A curve has polar equation $r = 1 + \cos\theta$ (a cardioid).

(a) Find r when $\theta = 0$ and $\theta = \frac{\pi}{2}$. [2]

(b) Find the area enclosed by the curve, using $A = \frac{1}{2}\int_0^{2\pi} (1+\cos\theta)^2\, d\theta$. [5]

3.3 Convert $r = \dfrac{2}{1+\cos\theta}$ to Cartesian form. [4]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **1.5 Polar coordinates** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/11 N25 —Q6 (polar curve / area)
- 9231/12 N25 —Q5 (polar coordinates)
- 9231/14 N25 —Q6 (polar area)

Solutions

2.1 $x = 2\cos\frac{\pi}{3} = 1$, $y = 2\sin\frac{\pi}{3} = \sqrt{3}$.

2.2 $A = \frac{1}{2}\displaystyle\int r^2\,d\theta$.

3.1 **B**. $r = 2\sin\theta \Rightarrow r^2 = 2r\sin\theta \Rightarrow x^2 + y^2 = 2y$, a circle.

3.2 (a) $\theta = 0$: $r = 2$; $\theta = \frac{\pi}{2}$: $r = 1$.
(b) $(1+\cos\theta)^2 = 1+2\cos\theta+\cos^2\theta = \frac{3}{2}+2\cos\theta+\frac{1}{2}\cos 2\theta$; $A = \frac{1}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{2\pi}$;
$= \frac{1}{2}\left(\frac{3}{2}\cdot 2\pi\right) = \frac{3\pi}{2}$.

3.3 $r + r\cos\theta = 2 \Rightarrow r = 2 - x$ (since $r\cos\theta = x$); $r^2 = (2-x)^2 \Rightarrow x^2 + y^2 = 4 - 4x + x^2$;
$y^2 = 4 - 4x$, i.e. $y^2 = -4(x-1)$.