1.3 Summation of series

Name: ____________ Class: ________ Date: __________ **Total: 14 marks**

Objective

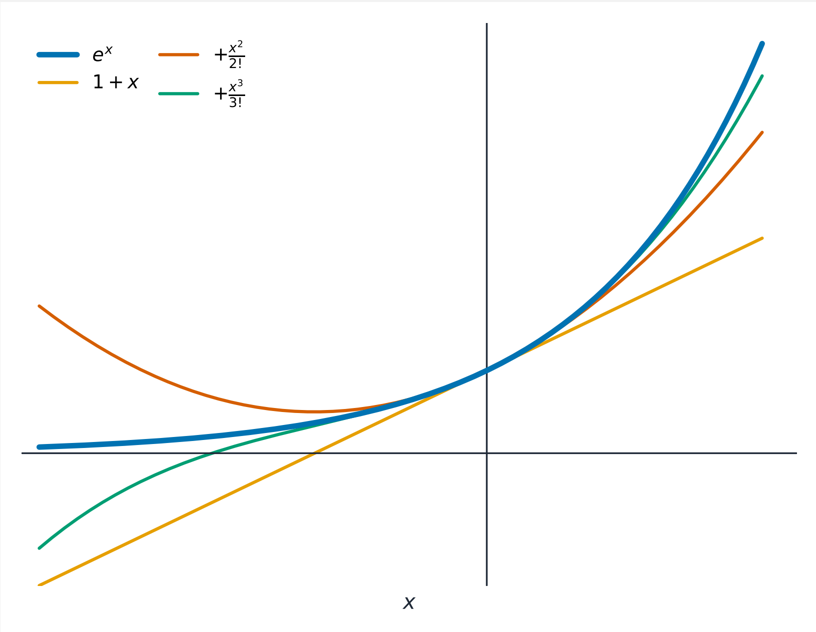
Build the skills to answer exam questions on the **summation of series** 级数求和—the standard results for $\sum r$, $\sum r^2$, $\sum r^3$, and the **method of differences** 差分法.

You must be able to:

- use $\sum r = \frac{1}{2}n(n+1)$, $\sum r^2 = \frac{1}{6}n(n+1)(2n+1)$, $\sum r^3 = \frac{1}{4}n^2(n+1)^2$
- sum a series by the method of differences (telescoping)
- decide convergence and find a sum to infinity

1 Worked examples

■ Standard results & differences



A finite sum builds from standard results; telescoping cancels all but the end terms

$$\sum_{r=1}^{n}(2r+1) = 2\cdot\tfrac{1}{2}n(n+1) + n = n(n+2).$$

Differences: if $u_r = f(r) - f(r+1)$ then $\sum_{r=1}^{n} u_r = f(1) - f(n+1)$.

2 Practice

2.1 Evaluate $\sum_{r=1}^{n} r^2$ as a formula in n. [1]

2.2 Find $\sum_{r=1}^{10}(3r-1)$. [2]

3 Exam-style questions

3.1 $\sum_{r=1}^{n} r^3$ equals: [1]

- **A** $\frac{1}{4}n^2(n+1)^2$
- **B** $\frac{1}{6}n(n+1)(2n+1)$
- **C** $\frac{1}{2}n(n+1)$
- **D** n^3

3.2 (a) Express $\dfrac{1}{r(r+1)}$ in partial fractions. [2]

(b) Hence find $\sum_{r=1}^{n} \dfrac{1}{r(r+1)}$ by the method of differences, and state its sum to infinity. [4]

3.3 Find $\sum_{r=1}^{n}(r^2 + r)$, giving your answer as a single factorised expression. [4]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **1.3 Summation of series** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/11 N25 —Q1 (summation)
- 9231/12 N25 —Q1 (method of differences)
- 9231/14 N25 —Q3 (series)

Solutions

2.1 $\frac{1}{6}n(n+1)(2n+1)$.

2.2 $3 \cdot \frac{1}{2}(10)(11) - 10 = 165 - 10 = 155$.

3.1 **A**. $\sum r^3 = \frac{1}{4}n^2(n+1)^2$.

3.2 (a) $\dfrac{1}{r(r+1)} = \dfrac{1}{r} - \dfrac{1}{r+1}$.

(b) $\displaystyle\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 1 - \frac{1}{n+1} = \frac{n}{n+1}$; as $n \to \infty$ the sum $\to 1$.

3.3 $\sum(r^2 + r) = \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$; $= \frac{1}{6}n(n+1)[(2n+1)+3]$; $= \frac{1}{6}n(n+1)(2n+4) = \frac{1}{3}n(n+1)(n+2)$.