1.2 Rational functions and graphs

Name: ____________ Class: ________ Date: __________ **Total: 15 marks**

KEY WORDS asymptotes 渐近线 • oblique 斜渐近线 • discriminant 判别式 • rational function 有理函数 • rational functions and graphs 有理函数与图像

Objective

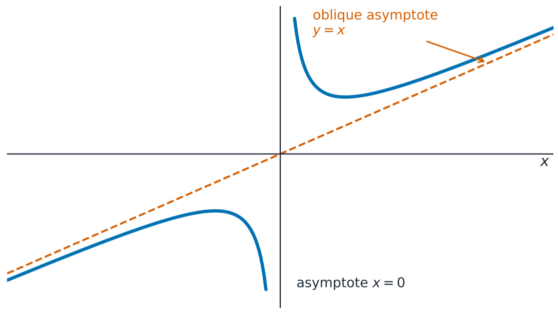
Build the skills to answer exam questions on **rational functions and graphs** 有理函数与图像 — **asymptotes** 渐近线 (including **oblique** 斜渐近线), the range via the **discriminant** 判别式, and related graphs.

You must be able to:

- find vertical and oblique asymptotes (divide out)
- find the set of values of a rational function using the discriminant
- relate $y = f(x)$ to $y = \frac{1}{f(x)}$, $y = |f(x)|$, $y^2 = f(x)$

1 Worked examples

■ Asymptotes & range



oblique asymptote when deg(num) = deg(den) + 1 (long division)

Far out the curve hugs the slant line; near a zero of the denominator it runs off along a vertical asymptote

$y = \dfrac{x^2+1}{x} = x + \dfrac{1}{x}$: vertical asymptote $x = 0$, oblique asymptote $y = x$.

Range of $y = \dfrac{x^2+1}{x}$: $yx = x^2 + 1 \Rightarrow x^2 - yx + 1 = 0$; real x needs $y^2 - 4 \geqslant 0$, so $y \leqslant -2$ or $y \geqslant 2$.

2 Practice

2.1 Write down the vertical asymptote of $y = \dfrac{2x+1}{x-3}$. [1]

2.2 Find the oblique asymptote of $y = \dfrac{x^2}{x-1}$. [2]

3 Exam-style questions

3.1 The curve $y = \dfrac{x^2 - 4}{x}$ has an oblique asymptote: [1]

- **A** $y = x$
- **B** $y = x - 4$
- **C** $y = 2x$
- **D** $y = -4$

3.2 A curve has equation $y = \dfrac{x^2 + 2}{x - 1}$.

(a) Find the equations of the two asymptotes. [3]

(b) Use the discriminant to find the set of values that y **cannot** take. [4]

3.3 The function is $y = \dfrac{x}{x^2 + 1}$. Find the set of values y can take. [4]

Solutions

2.1 $x = 3$.

2.2 $\dfrac{x^2}{x - 1} = x + 1 + \dfrac{1}{x - 1}$, so oblique asymptote $y = x + 1$.

3.1 A. $\dfrac{x^2 - 4}{x} = x - \dfrac{4}{x}$, so the oblique asymptote is $y = x$.

3.2 (a) vertical $x = 1$; divide: $\dfrac{x^2 + 2}{x - 1} = x + 1 + \dfrac{3}{x - 1}$, oblique $y = x + 1$.
(b) $y(x-1) = x^2+2 \Rightarrow x^2 - yx + (y+2) = 0$; real x needs $y^2 - 4(y+2) \geqslant 0 \Rightarrow y^2 - 4y - 8 \geqslant 0$; roots $y = 2 \pm 2\sqrt{3}$; so y cannot take values in $2 - 2\sqrt{3} < y < 2 + 2\sqrt{3}$.

3.3 $y(x^2 + 1) = x \Rightarrow yx^2 - x + y = 0$; for real x (with $y \neq 0$), $1 - 4y^2 \geqslant 0$; $-\frac{1}{2} \leqslant y \leqslant \frac{1}{2}$.