Past-paper walk-through

Non-parametric tests ▪ 4.4

eXercise

Questions in this set

- 9231/41 N25 Q4 ▪ 5 marks
- 9231/42 N25 Q1 ▪ 6 marks
- 9231/44 N25 Q6 ▪ 7 marks
- 9231/41 J25 Q5 ▪ 10 marks
- 9231/43 J25 Q4 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

9231/41 N25 ▪ Q4 ▪ 5 marks

A researcher believes that the median m of a population has changed from its known previous value m_0. The researcher collects a random sample of size 28. She ranks the data and calculates a test statistic T using the Wilcoxon signed-rank test. The conclusion of the test carried out at a 1% significance level is that there is not sufficient evidence to support her belief. Using a normal approximation, find the least possible value of T. **[5]**

Solution 4

Mark scheme

- Normal approximation: mean $= \frac{1}{4} \cdot 28 \cdot 29 = 203$, variance $= \frac{1}{24} \cdot 28 \cdot 29 \cdot 57 = 1928.5$. Two-tailed 1% test, critical $z = -2.576$. No rejection requires $\dfrac{T + 0.5 - 203}{\sqrt{1928.5}} > -2.576$, so $T > 89.4$. The least possible value is $T = 90$.

Question 1

9231/42 N25 ▪ Q1 ▪ 6 marks

A large company claims that the median salary of its employees is $32500. The salaries ($) of 15 randomly selected employees are: 18750, 30500, 125000, 42500, 25000, 26000, 52500, 23000, 27500, 19500, 25500, 33000, 30000, 21500, 29000.

(a) Explain why a Wilcoxon signed-rank test may not be appropriate to test the company's claim in this case. **[1]**

(b) Carry out a sign test at the 10% significance level to investigate the company's claim. **[5]**

Solution 1

(a)

Mark scheme

- The Wilcoxon signed-rank test assumes a symmetric distribution; the salaries are strongly skewed (e.g. \$125000 is a large outlier), so the symmetry assumption is not reasonable.

Solution 1

(b)

Mark scheme

- H_0: median $= 32500$, H_1: median $\neq 32500$. Signs of (salary -32500): 4 positive, 11 negative. Under H_0 the number of $+$ signs $\sim B(15, 0.5)$; $P(X \leqslant 4) = 0.0592 > 0.05$ (two-tailed at 10

Question 6

9231/44 N25 ▪ Q6 ▪ 7 marks

Students of the same age from two schools, school A and school B, take a large number of quizzes throughout the year and are each awarded a mark out of 1000. The marks of 123 students in school A and 147 students in school B are ranked from lowest (rank 1) to highest (rank 270). The sum of the ranks of the students from school A is 15355.

(a) Carry out a Wilcoxon rank-sum test at the 5% significance level to investigate whether there is a difference in average marks between the students in school A and school B. **[6]**

(b) State an assumption that is required for the Wilcoxon rank-sum test to be valid. **[1]**

Solution 6

(a) Mark scheme

- H_0: the population median mark for school A equals that for school B; H_1: they are not equal. Mean $= \frac{1}{2} \cdot 123 \cdot (123 + 147 + 1) = 16666.5$; variance $= \frac{1}{12} \cdot 123 \cdot 147 \cdot (123 + 147 + 1) = 408329.25$. With continuity correction, $z = \dfrac{15355 + 0.5 - 16666.5}{\sqrt{408329.25}} = -2.05$. Two-tailed 5% test: since $-2.05 < -1.96$, reject H_0: there is sufficient evidence of a difference in average marks between the students in school A and school B.

Solution 6

(b)

Mark scheme

- The (underlying) distributions are symmetrical / of similar shape (or there are no repeated marks).

Question 5

9231/41 J25 ▪ Q5 ▪ 10 marks

Two jigsaw puzzles have the same number of pieces with identical shapes but have different pictures printed on them (a seaside picture and a cartoon picture). A researcher believes that children will complete the cartoon puzzle more quickly. To test this belief, 10 children are randomly selected and the time taken in seconds for each child to complete each puzzle is recorded. Children *A*–*J*: Seaside: 182, 130, 193, 181, 192, 204, 184, 192, 180, 189; Cartoon: 161, 111, 195, 159, 202, 200, 168, 165, 145, 160.

(a) Carry out a Wilcoxon matched-pairs signed-rank test at the 5% significance level to test the researcher's belief. **[6]**

(b) Show that using a paired-sample sign test at the 5% significance level would result in the opposite conclusion to that found in part (a). **[3]**

Question 5

(c) It was later discovered that the experiment had been conducted such that each child completed the seaside puzzle first followed by the cartoon puzzle. Comment on the validity of using this experiment to test the researcher's belief. **[1]**

Question 5

Child	A	B	C	D	E	F	G	H	I	J
Seaside	182	130	193	181	192	204	184	192	180	189
Cartoon	161	111	195	159	202	200	168	165	145	160

Solution 5

(a)

Mark scheme

- H_0: $m_s = m_c$; H_1: $m_s > m_c$ (population medians, seaside vs cartoon). Differences (seaside − cartoon): $21, 19, -2, 22, -10, 4, 16, 27, 35, 29$. Signed ranks: $6, 5, -1, 7, -3, 2, 4, 8, 10, 9$, so $P = 51$, $Q = 4$ and $T = 4$. For $n = 10$, one-tailed 5%, critical value $= 10$. Since $4 \leqslant 10$, reject H_0: there is sufficient evidence that children, on average, complete the cartoon puzzle more quickly than the seaside puzzle (supporting the researcher's belief).

Solution 5

(b)

Mark scheme

- Sign test: 8 differences are positive and 2 are negative. Under H_0, $X \sim \mathrm{B}(10, 0.5)$, and $\mathrm{P}(X \leqslant 2) = 0.0547$. Since $0.0547 > 0.05$, do not reject H_0 (not significant). In part (a) H_0 was rejected but here H_0 is accepted, so the opposite conclusion is reached.

Solution 5

(c)

Mark scheme

- Because every child completes the seaside puzzle first, the order is a confounding factor: children may complete the second (cartoon) puzzle faster through practice (or slower through tiredness), so the experiment does not fairly test the researcher's belief.

Question 4

9231/43 J25 ▪ Q4 ▪ 8 marks

A researcher claims that older people take longer to react to a sudden loud noise than younger people. To investigate this, the researcher randomly selects 6 people over 50 years old and 8 people under 25 years old and records their reaction times, in milliseconds, to a sudden loud noise. Over 50: 198, 212, 217, 229, 235, 242. Under 25: 178, 181, 183, 192, 203, 209, 223, 231. Carry out a Wilcoxon rank-sum test at the 5% significance level to test the researcher's claim. **[8]**

Solution 4

Mark scheme

- H_0: the population medians for under-25s and over-50s are equal; H_1: the population median time for under-25s is less than that for over-50s. Ranking all 14 times 1–14, the over-50 sample (size 6) has ranks $5, 8, 9, 11, 13, 14$, so $R_6 = 60$; the complementary sum is $6(6+8+1) - 60 = 30$, giving $W = \min(60, 30) = 30$. Critical value for $n = 6$, $m = 8$ is 31. Since $30 \leqslant 31$, reject H_0: there is sufficient evidence that older people take longer to react (supporting the researcher's claim).