Past-paper walk-through

Inference using normal and t-distributions ▪ 4.2

eXercise

Questions in this set

- 9231/41 N25 Q1 ▪ 6 marks
- 9231/41 N25 Q3 ▪ 4 marks
- 9231/41 N25 Q6 ▪ 9 marks
- 9231/42 N25 Q2 ▪ 7 marks
- 9231/42 N25 Q5 ▪ 9 marks
- 9231/44 N25 Q1 ▪ 6 marks
- 9231/44 N25 Q3 ▪ 10 marks
- 9231/41 J25 Q1 ▪ 7 marks
- 9231/41 J25 Q4 ▪ 7 marks
- 9231/43 J25 Q2 ▪ 7 marks

Questions in this set

- 9231/43 J25 Q5 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 1

9231/41 N25 ▪ Q1 ▪ 6 marks

A group of 10 school children are asked to estimate the size of an angle θ in a given acute angled triangle. These estimates, in degrees, are as follows: 84, 85, 77, 85, 84, 87, 86, 88, 83, 85.

(a) Stating any assumptions you make, calculate a 95% confidence interval for θ. **[5]**

(b) Give a reason why the assumptions made in part (a) may not be appropriate in this case. **[1]**

Solution 1

(a) Mark scheme

- $\bar{x} = 84.4$; $s^2 = \frac{1}{9}\left(71314 - \frac{844^2}{10}\right) = \frac{134}{15} = 8.933$. Using the t-value 2.262 (9 d.f.): $84.4 \pm 2.262\sqrt{\frac{8.933}{10}}$, giving the 95% CI $[82.3, 86.5]$. Assumption: the distribution of estimates is normal (and the estimates are a random, independent sample).

Solution 1

(b)

Mark scheme

- The population is unlikely to be normal as θ is close to a right angle / the data are skewed; or the estimates may not be independent (e.g. due to collusion); or there is no indication the sample is random.

Question 3

9231/41 N25 ▪ Q3 ▪ 4 marks

A random sample of 10 newborn baby boys is taken and their masses in kg are recorded. From this sample, the population standard deviation of all newborn baby boys is estimated as 0.6 kg. A random sample of 5 newborn baby girls is taken and their masses in kg are recorded as follows: 3.9, 3.1, 2.9, 3.1, 3.6. It is assumed that the masses of newborn baby boys and girls have the same population standard deviation, σ kg. By pooling the two samples, calculate an estimate of σ. **[4]**

Solution 3

Mark scheme

- Boys: $\sum(x-\bar{x})^2 = 9 \times 0.36 = 3.24$. Girls ($\sum x = 16.6$, $\sum x^2 = 55.8$): $\sum(x-\bar{x})^2 = 55.8 - \frac{16.6^2}{5} = 0.688$. Pooled variance $= \dfrac{3.24 + 0.688}{10+5-2} = 0.302$, so $s = 0.550$.

Question 6

9231/41 N25 ▪ Q6 ▪ 9 marks

Nine athletes in a club have a new coach. The coach adopts a new training programme which he believes will reduce the race times of these athletes. Each athlete completes a 1500 m time trial before and after completing the new training programme. Their times, in seconds (s), are recorded (athletes *A*–*I*: before 250, 251, 252, 267, 276, 291, 310, 320, 335; after 245, 251, 253, 261, 275, 293, 302, 313, 320).

(a) Carry out a paired t-test at the 5% significance level to test the coach's belief. **[7]**

(b) Further research suggests that the effects of the training programme tend to reduce the times of the slower athletes by more than those of the faster athletes. Suggest a reason why the paired t-test used in part (a) may not have been an appropriate test in this case. **[1]**

(c) Suggest a suitable alternative test that could have been used instead of a paired t-test. **[1]**

Question 6

Athlete	A	B	C	D	E	F	G	H	I
Time before training (s)	250	251	252	267	276	291	310	320	335
Time after training (s)	245	251	253	261	275	293	302	313	320

Solution 6

(a) Mark scheme

- H_0: $\mu_B = \mu_A$; H_1: $\mu_B > \mu_A$ (population means, before vs after). Differences (before − after): $5, 0, -1, 6, 1, -2, 8, 7, 15$; $\bar{d} = \frac{39}{9}$, $s_d^2 = \frac{1}{8}\left(405 - \frac{39^2}{9}\right) = 29.5$. $t = \dfrac{39/9}{\sqrt{29.5/9}} = 2.393$. With 8 d.f., critical $t = 1.860$. Since $2.393 > 1.860$, reject H_0: there is sufficient evidence that the new training programme reduces times.

Solution 6

(b)

Mark scheme

- The population of differences may not be normally distributed / may not be symmetrical (since the reduction depends on the athlete's speed).

Solution 6

(c)

Mark scheme

- A Wilcoxon matched-pairs signed-rank test (or a paired-sample sign test).

Question 2

9231/42 N25 ▪ Q2 ▪ 7 marks

A factory produces packets of biscuits. The total mass of biscuits in a packet has a normal distribution with mean μ. A random sample of 12 packets is taken and the mass of the contents of each packet, x g, is recorded, giving $\sum x = 2390$ and $\sum x^2 = 476117$.

(a) Find a 99

(b) A test of the null hypothesis $\mu = k$ is carried out on this sample using a 5

Solution 2

(a)

Mark scheme

- $\bar{x} = 199.17$; $s^2 = \frac{1}{11}\left(476117 - \frac{2390^2}{12}\right) = 9.879$. With $t_{0.005,\,11} = 3.106$: CI $= 199.17 \pm 3.106 \cdot \frac{3.143}{\sqrt{12}} = (196.3,\ 202.0)$.

Solution 2

(b)

Mark scheme

- Not supporting $\mu < k$ requires $\dfrac{\bar{x} - k}{s/\sqrt{12}} \geqslant -t_{0.05,\,11} = -1.796$, so $k \leqslant \bar{x} + 1.796 \cdot \frac{s}{\sqrt{12}} = 200.8$. Greatest $k = 200.8$.

Question 5

9231/42 N25 ▪ Q5 ▪ 9 marks

An engineer is comparing the tensile strengths of steel rods made from two machines, A and B, randomly selecting 8 rods from A and 6 from B. Machine A: 402, 403, 415, 412, 409, 407, 406, 410. Machine B: 401, 398, 395, 397, 410, 405. Assuming the two distributions are normal with the same population variance, use a t-test at the 5% significance level to test whether there is any difference in the mean tensile strengths of steel rods from the two machines. **[9]**

Solution 5

Mark scheme

- $\bar{x}_A = 408$, $\bar{x}_B = 401$; $\sum(x - \bar{x})^2 = 136$ (A) and 158 (B). Pooled $s_p^2 = \frac{136+158}{12} = 24.5$. $t = \dfrac{408 - 401}{\sqrt{24.5}\sqrt{\frac{1}{8} + \frac{1}{6}}} = 2.62$. With $\nu = 12$, $t_{0.025} = 2.179$; since $2.62 > 2.179$, reject H_0 — there is evidence of a difference in mean tensile strengths.

Question 1

9231/44 N25 ▪ Q1 ▪ 6 marks

Local residents are concerned about the speed of cars travelling through their village. They record the speed, in km h^{-1}, of a random sample of 13 cars travelling through their village. The recorded speeds are as follows: 40, 53, 59, 42, 43, 48, 62, 67, 46, 82, 66, 45, 70.

(a) Construct a 90% confidence interval for the population mean speed of cars passing through the village. **[5]**

(b) State an assumption that is necessary for the confidence interval found in part (a) to be valid. **[1]**

Solution 1

(a) Mark scheme

- $\sum x = 723$, $\sum x^2 = 42261$ (so $\bar{x} = 55.6$). Unbiased estimate $s^2 = \frac{1}{12}\left(42261 - \frac{723^2}{13}\right) = \frac{2222}{13} = 170.9231$ ($s = 13.07$). With the t-value 1.782 (12 d.f., 90%): $\frac{723}{13} \pm 1.782\sqrt{\frac{170.9231}{13}}$, giving the 90% CI $[49.2, 62.1]$.

Solution 1

(b)

Mark scheme

- The population (of car speeds) is normally distributed.

Question 3

9231/44 N25 ▪ Q3 ▪ 10 marks

Two different types of juice extractor, machine X and machine Y, are being compared. The manufacturer claims that machine Y extracts more juice per orange on average than machine X. A random sample of 20 oranges is selected, paired by similar size and mass and numbered 1 to 10; one orange from each pair is randomly allocated to machine X and the other to machine Y. The amount of juice, in ml, extracted is recorded (pairs 1–10): machine X: 65, 73, 58, 61, 72, 79, 64, 65, 69, 71; machine Y: 68, 72, 64, 63, 75, 82, 63, 63, 72, 74.

(a) Use a t-test to test the manufacturer's claim at the 1% significance level. **[7]**

(b) State an assumption required for the test in part (a) to be valid. **[1]**

(c) The manufacturer notices that the amount of juice extracted from the oranges in pair 4 were recorded incorrectly. In fact, the amount of juice extracted from machine X

Question 3

was 60 ml and from machine Y was 62 ml. Explain, with justification, whether the conclusion of the test in part (a) remains the same. **[2]**

Question 3

	pair									
	1	2	3	4	5	6	7	8	9	10
machine X	65	73	58	61	72	79	64	65	69	71
machine Y	68	72	64	63	75	82	63	63	72	74

Solution 3

(a) Mark scheme

- H_0: $\mu_Y = \mu_X$; H_1: $\mu_Y > \mu_X$ (population means). Differences $(Y - X)$: $3, -1, 6, 2, 3, 3, -1, -2, 3, 3$; $\sum d = 19$, $s_d^2 = \frac{1}{9}\left(91 - \frac{19^2}{10}\right) = 6.1$.
  $t = \dfrac{19/10}{\sqrt{6.1/10}} = 2.43$. With 9 d.f., critical $t = 2.821$. Since $2.43 < 2.821$, do not reject H_0: there is insufficient evidence to support the manufacturer's claim (that machine Y extracts more juice on average).

Solution 3

(b)

Mark scheme

- The population of differences is normally distributed.

Solution 3

(c)

Mark scheme

- For pair 4 the new difference is $62 - 60 = 2$, the same as the original $63 - 61 = 2$, so the set of differences is unchanged. Hence the test statistic and the conclusion remain the same.

Question 1

9231/41 J25 ▪ Q1 ▪ 7 marks

The manager of a hardware store is interested in whether there is a difference in the amount spent per customer on weekdays ($\$x$) compared to weekends ($\$y$). Random samples of 120 customers on weekdays and 80 customers on weekends are taken and the amount spent by each customer is recorded. The results are summarised as follows: $\sum x = 10470$, $\sum(x-\bar{x})^2 = 12283$, $\sum y = 6560$, $\sum(y-\bar{y})^2 = 13520$. Test at the 1% significance level whether there is a difference in the mean amount spent per customer on weekdays compared to weekends. You should not assume that the population variances of the amounts spent on weekdays and weekends are equal. **[7]**

Solution 1

Mark scheme

- H_0: $\mu_x = \mu_y$; H_1: $\mu_x \neq \mu_y$ (population means). Unbiased estimates $s_x^2 = \frac{12283}{119} = 103.218$, $s_y^2 = \frac{13520}{79} = 171.139$, so $\frac{s_x^2}{120} + \frac{s_y^2}{80} = 2.999$.
  $z = \dfrac{\frac{10470}{120} - \frac{6560}{80}}{\sqrt{2.999}} = \dfrac{87.25 - 82}{1.732} = 3.03$. Two-tailed 1% test, critical $z = 2.576$.
  Since $3.03 > 2.576$, reject H_0: there is sufficient evidence that the mean spend per customer is different on weekends compared to weekdays.

Question 4

9231/41 J25 ▪ Q4 ▪ 7 marks

A researcher is interested in whether there is a difference between two schools in students' aptitude for English. She randomly chooses 10 students from school X and 8 students of a similar age from school Y to take a written English test. The scores for school X (x) and school Y (y) are summarised as follows: $\sum x = 612$, $\sum x^2 = 40104$, $\sum y = 444$, $\sum y^2 = 27460$. You should assume that the two distributions are normal and have the same population variance.

(a) Find a 95% confidence interval for the difference in the mean scores for students from school X and students from school Y in the written English test. **[6]**

(b) Use the confidence interval you found in part (a) to explain why there is insufficient evidence at a 5% significance level to suggest that the English scores of students from school X and students from school Y are different. **[1]**

Solution 4

(a) Mark scheme

- $\sum(x-\bar{x})^2 = 40104 - \frac{1}{10}\cdot 612^2 = 2649.6$; $\sum(y-\bar{y})^2 = 27460 - \frac{1}{8}\cdot 444^2 = 2818$. Pooled $s_p^2 = \dfrac{2649.6 + 2818}{10+8-2} = 341.725$. With $\bar{x} = 61.2$, $\bar{y} = 55.5$ and $t = 2.120$ (16 d.f.): $(61.2 - 55.5) \pm 2.120\sqrt{s_p^2\left(\frac{1}{10}+\frac{1}{8}\right)} = 5.7 \pm 2.120 \times 8.7686$, giving the 95% CI $[-12.9, 24.3]$.

Solution 4

(b)

Mark scheme

- Because zero lies inside the confidence interval, so there is insufficient evidence of a difference at the 5\% level.

Question 2

9231/43 J25 ▪ Q2 ▪ 7 marks

A farmer is investigating whether using a new fertiliser will increase the yield of tomato plants. The farmer selects 40 tomato plants at random and gives them the new fertiliser; the crop mass, x kg, of each is recorded. A further 60 tomato plants are given a standard fertiliser; the crop mass, y kg, of each is recorded. The results are summarised as follows: $\sum x = 168$, $\sum x^2 = 720$, $\sum y = 228$, $\sum y^2 = 900$. Find a 90% confidence interval for the difference in mean crop mass associated with each type of fertiliser. **[7]**

Solution 2

Mark scheme

- Unbiased estimates: $s_x^2 = \frac{1}{39}\left(720 - \frac{168^2}{40}\right) = 0.36923$, $s_y^2 = \frac{1}{59}\left(900 - \frac{228^2}{60}\right) = 0.56949$. For the difference of means, $s^2 = \frac{s_x^2}{40} + \frac{s_y^2}{60} = 0.0187$. With $\bar{x} = 4.2$, $\bar{y} = 3.8$ and $z = 1.645$: $\left(\frac{168}{40} - \frac{228}{60}\right) \pm 1.645\sqrt{0.0187} = 0.4 \pm 0.225$, giving the 90% CI $[0.175, 0.625]$.

Question 5

9231/43 J25 ▪ Q5 ▪ 8 marks

A doctor is investigating the concentration of blood glucose in patients at risk of developing type 2 diabetes. The doctor claims that a particular intervention reduces the concentration by more than k units on average. A group of 8 at-risk patients is selected at random and each follows the intervention for six months. The blood glucose concentrations before and after the intervention (patients A–H): Before: 183, 165, 172, 165, 143, 176, 161, 153; After: 164, 148, 164, 149, 134, 153, 155, 148.

(a) Use a t-test at the 5% significance level to find the range of values of k for which the result of the test is to reject the null hypothesis. **[7]**

(b) State an assumption necessary for the test in part (a) to be valid. **[1]**

Question 5

Patient	A	B	C	D	E	F	G	H
Before	183	165	172	165	143	176	161	153
After	164	148	164	149	134	153	155	148

Solution 5

(a) Mark scheme

- H_0: $\mu_B - \mu_A = k$; H_1: $\mu_B - \mu_A > k$ (before − after). Differences d: $19, 17, 8, 16, 9, 23, 6, 5$; $\sum d = 103$, $\sum d^2 = 1641$, so $\bar{d} = 12.875$ and $s_d^2 = \frac{1}{7}\left(1641 - \frac{103^2}{8}\right) = 44.982$. The test statistic is $t = \dfrac{12.875 - k}{\sqrt{s_d^2/8}}$; with 7 d.f. the one-tailed 5% critical value is 1.895. Reject H_0 when $\dfrac{12.875 - k}{\sqrt{s_d^2/8}} \geqslant 1.895$, i.e. when $0 \leqslant k \leqslant 8.38$.

Solution 5

(b)

Mark scheme

- The population of differences is normally distributed.