Exercise walk-through

Inference using normal and t-distributions ▪ 4.2

eXercise

You must be able to

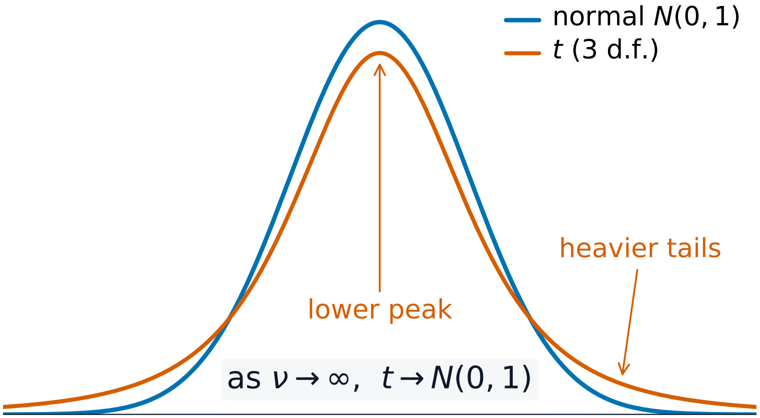
- find a confidence interval $\bar{x} \pm t\frac{s}{\sqrt{n}}$ (with $n-1$ d.f.)
- carry out a one-sample t-test
- carry out a two-sample / paired t-test (pooled variance where needed)

Method — Small-sample interval

$n = 10$**,** $\bar{x} = 50$**,** $s = 4$**,** 95% **(**$t = 2.262$**,** 9 **d.f.):**

$$50 \pm 2.262\frac{4}{\sqrt{10}} = 50 \pm 2.86 = (47.1,\ 52.9).$$

Method — Small-sample interval



The t-distribution curve sitting lower and wider than the normal

Practice 2.1 ▪ 1 mark

State the number of degrees of freedom for a one-sample t-test with $n = 12$.

Solution 2.1

Worked steps

$n - 1 = 11$ **B1**.

Practice 2.2 ▪ 1 mark

State when a t-distribution is used instead of the normal.

Solution 2.2

Method: Small-sample interval

Worked steps

when the sample is small and the population variance is unknown **B1**.

Exam-style 3.1 ▪ 1 mark

A 95% confidence interval based on a small sample uses a t value that is, compared with 1.96:

A smaller

B larger

C equal

D zero

Solution 3.1

Method: Small-sample interval

A smaller

B larger ✓

C equal

D zero

Worked steps

B. The small-sample t value is larger than 1.96.

Exam-style 3.2 ▪ 3 marks

A sample of 8 measurements has $\bar{x} = 20.5$ and $s = 1.2$.

(a) Find a 95% confidence interval for the population mean (use $t = 2.365$, 7 d.f.).

(b) State one assumption needed for this interval to be valid.

Solution 3.2

Method: Small-sample interval

Worked steps

(a) $20.5 \pm 2.365\dfrac{1.2}{\sqrt{8}} = 20.5 \pm 2.365(0.4243) = 20.5 \pm 1.00$ **M1** **M1**; $(19.5,\ 21.5)$ **A1**.

(b) the population is (approximately) normally distributed **B1**.

Exam-style 3.3 ▪ 5 marks

A machine should produce rods of mean length 25 cm. A sample of 6 rods has $\bar{x} = 25.4$ cm and $s = 0.5$ cm. Test at the 5% level (two-tailed, $t_{\text{crit}} = 2.571$) whether the mean differs from 25.

Solution 3.3

Method: Small-sample interval

Worked steps

$H_0: \mu = 25,\ H_1: \mu \neq 25$ **B1**; $t = \dfrac{25.4 - 25}{0.5/\sqrt{6}} = \dfrac{0.4}{0.2041} = 1.96$ **M1** **M1**; compare with 2.571 **M1**; since $1.96 < 2.571$, do not reject H_0: no evidence the mean differs from 25 **A1**.