Exercise walk-through

Continuous random variables ▪ 4.1

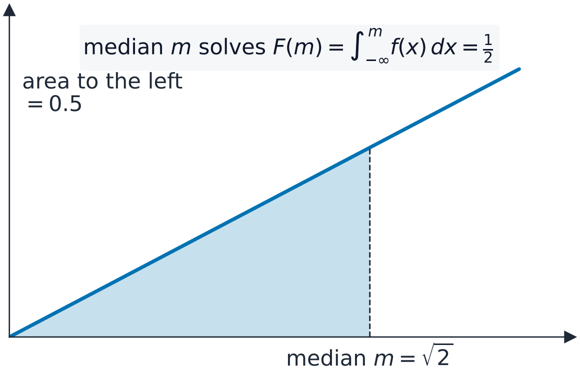
eXercise

You must be able to

- find a constant / probability from a pdf (possibly piecewise)
- find $E(X)$, $\mathrm{Var}(X)$ and $E(g(X))$ by integration
- find $F(x)$ and use it for the median and percentiles

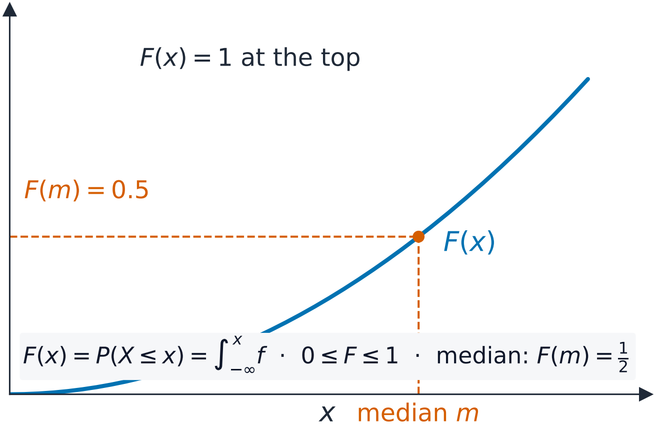
Method — pdf, cdf and median

$f(x) = \frac{1}{2}x$ **on** $[0, 2]$**:** $F(x) = \frac{1}{4}x^2$; median $\frac{1}{4}m^2 = 0.5 \Rightarrow m = \sqrt{2}$.



A density with the area left of the median shaded as one half

Method — pdf, cdf and median



A cumulative curve rising from 0 to 1, median read off at 0.5

Practice 2.1 ▪ 2 marks

For $f(x) = \frac{1}{2}x$ on $[0, 2]$, find $E(X)$.

Solution 2.1

Worked steps

$$E(X) = \int_0^2 x \cdot \tfrac{1}{2} x \, dx = \left[\tfrac{x^3}{6} \right]_0^2 = \frac{4}{3} \quad \boxed{\textbf{M1}}\boxed{\textbf{A1}}.$$

Practice 2.2 ▪ 1 mark

Write the relationship between $f(x)$ and $F(x)$.

Solution 2.2

Worked steps

$f(x) = F'(x)$ **B1**.

Exam-style 3.1 ▪ 1 mark

The median of a continuous variable solves:

A $f(m) = 0.5$

B $F(m) = 0.5$

C $E(X) = m$

D $F(m) = 1$

Solution 3.1

A $f(m) = 0.5$

B $F(m) = 0.5$ ✓

C $E(X) = m$

D $F(m) = 1$

Worked steps

B. The median solves $F(m) = 0.5$.

Exam-style 3.2 ▪ 3 marks

A continuous variable has $f(x) = kx^2$ for $0 \leqslant x \leqslant 3$ (and 0 elsewhere).
(a) Find k.
(b) Find $E(X)$.

Solution 3.2

Worked steps

(a) $\int_0^3 kx^2\,dx = \left[\frac{kx^3}{3}\right]_0^3 = 9k = 1 \Rightarrow k = \frac{1}{9}$ M1 M1 A1.

(b) $E(X) = \int_0^3 x \cdot \frac{1}{9}x^2\,dx = \frac{1}{9}\left[\frac{x^4}{4}\right]_0^3 = \frac{1}{9} \cdot \frac{81}{4} = \frac{9}{4}$ M1 M1 A1.

Exam-style 3.3 ▪ 2 marks

A variable has cdf $F(x) = \dfrac{x^2}{16}$ for $0 \leqslant x \leqslant 4$.

(a) Find the pdf $f(x)$.

(b) Find the median.

Solution 3.3

Worked steps

(a) $f(x) = F'(x) = \dfrac{x}{8}$ for $0 \leqslant x \leqslant 4$ **M1** **A1**.

(b) $\dfrac{m^2}{16} = 0.5 \Rightarrow m^2 = 8 \Rightarrow m = 2\sqrt{2} = 2.83$ **M1** **A1**.