Past-paper walk-through

Momentum ▪ 3.6

eXercise

Questions in this set

- 9231/31 N25 Q1 ▪ 4 marks
- 9231/32 N25 Q5 ▪ 9 marks
- 9231/34 N25 Q7 ▪ 6 marks
- 9231/31 J25 Q6 ▪ 9 marks
- 9231/33 J25 Q6 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 1

9231/31 N25 ▪ Q1 ▪ 4 marks

Two uniform smooth spheres A and B of equal radii have masses $4m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides directly with sphere B. After the collision, the momentum of A is three times the momentum of B. Find the value of the coefficient of restitution e. **[4]**

Solution 1

Mark scheme

- Conservation of momentum: $4mv_A + mv_B = 4mu$. NEL: $-v_A + v_B = eu$. The condition (momentum of A is three times that of B, i.e. $4v_A = 3v_B$) gives $v_A = \frac{3}{4}u$, $v_B = u$. Then $eu = v_B - v_A = u - \frac{3}{4}u = \frac{1}{4}u$, so $e = \frac{1}{4}$.

Question 5

9231/32 N25 ▪ Q5 ▪ 9 marks

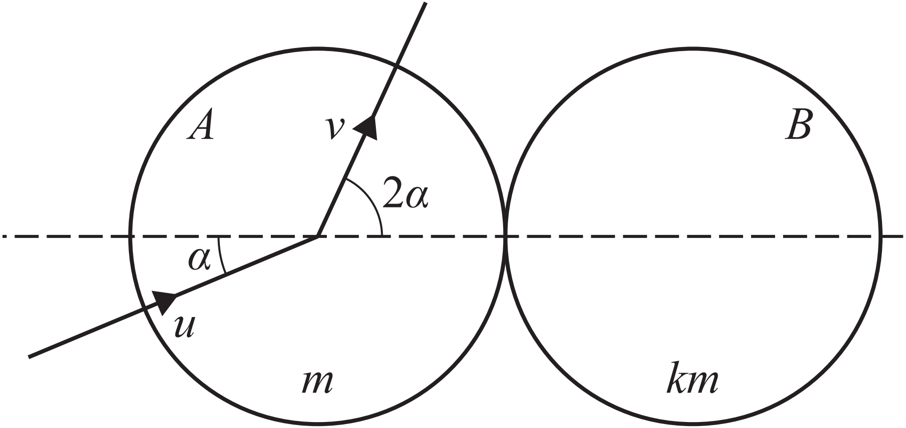
Two uniform smooth spheres, A and B, of equal radii are on a horizontal surface. They have masses m and km respectively. Sphere A is moving with speed u at an angle α with the line of centres when it collides with sphere B, which is stationary. Immediately after the collision, sphere A moves with speed v at an angle 2α with the line of centres (see diagram). It is given that $\tan\alpha = \frac{3}{4}$.

(a) Find v in terms of u. **[2]**

(b) Find the coefficient of restitution between the spheres in terms of k. **[4]**

(c) Find the range of possible values of k. **[3]**

Question 5



Solution 5

(a)

Mark scheme

- Perpendicular (to line of centres) component of A is unchanged:
  $u \sin\alpha = v \sin 2\alpha = 2v \sin\alpha\cos\alpha \Rightarrow v = \dfrac{u}{2\cos\alpha} = \dfrac{5u}{8}$ (since $\cos\alpha = \frac{4}{5}$).

Solution 5

(b) Mark scheme

- Along the centres, momentum: $u\cos\alpha = v\cos 2\alpha + kw$ gives $w = \frac{5u}{8k}$. With $\cos\alpha = \frac{4}{5}$, $\cos 2\alpha = \frac{7}{25}$, $v = \frac{5u}{8}$: $e = \dfrac{w - v\cos 2\alpha}{u\cos\alpha} = \dfrac{25-7k}{32k}$.

Solution 5

(c)

Mark scheme

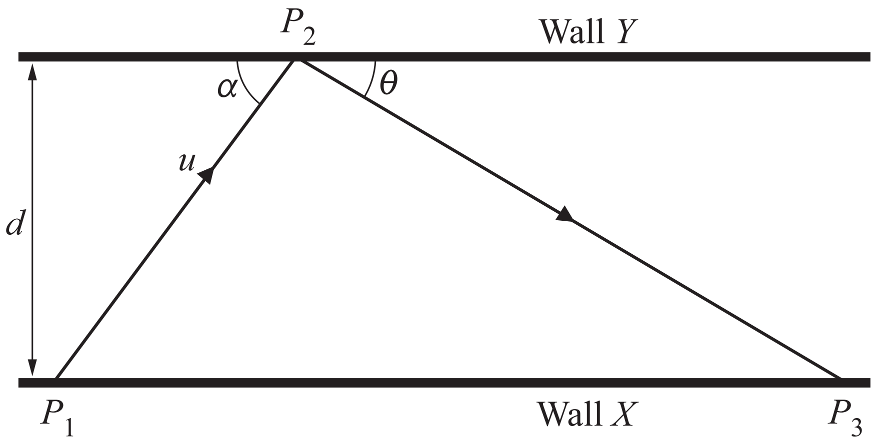
- Require $0 \leqslant e \leqslant 1$ with $k > 0$: $e \geqslant 0 \Rightarrow k \leqslant \frac{25}{7}$; $e \leqslant 1 \Rightarrow 25 - 7k \leqslant 32k \Rightarrow k \geqslant \frac{25}{39}$. So $\frac{25}{39} \leqslant k \leqslant \frac{25}{7}$.

Question 7

9231/34 N25 ▪ Q7 ▪ 6 marks

X and Y are two fixed smooth vertical walls on a smooth horizontal surface. The walls are parallel and at a distance d apart. The points P_1, P_2 and P_3 all lie on the surface. A particle Q is projected horizontally from the point P_1 on Wall X with speed u, and moves along the surface. The particle Q strikes Wall Y at the point P_2. Immediately before the collision, the direction of motion of Q makes an angle α with Wall Y, where $\sin\alpha = \frac{4}{5}$. Immediately after the collision, the direction of motion of Q makes an angle θ with Wall Y. The particle Q then strikes Wall X at the point P_3 (see diagram). The time that it takes Q to travel the distance P_1P_2 is T. The time that it takes Q to travel the distance P_2P_3 is kT. Find, in terms of k, the coefficient of restitution between Q and Wall Y. **[6]**

Question 7



Solution 7

Mark scheme

- Perpendicular to Wall Y (restitution): $eu\sin\alpha = v\sin\theta$, where v is the speed after the collision. Distances: $P_1P_2 = uT$, $P_2P_3 = vkT$; and $\sin\alpha = \dfrac{d}{P_1P_2} = \dfrac{d}{uT}$, $\sin\theta = \dfrac{d}{P_2P_3} = \dfrac{d}{vkT}$. Substituting: $eu \cdot \dfrac{d}{uT} = v \cdot \dfrac{d}{vkT}$, i.e. $\dfrac{ed}{T} = \dfrac{d}{kT}$, so $e = \dfrac{1}{k}$.

Question 6

9231/31 J25 ▪ Q6 ▪ 9 marks

Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere A is moving in a straight horizontal line with speed u, and sphere B is stationary. Sphere A collides directly with B, and they both then move in the same direction with speeds v_A and v_B respectively. After the collision, the kinetic energy of B is $\frac{9}{2}$ times the kinetic energy of A.

(a) Show that $v_B = \frac{6}{5}u$. **[3]**

(b) Sphere B then collides with a fixed vertical barrier. Immediately before the collision, the direction of motion of B makes an angle α with the barrier. Immediately after the collision, the direction of motion of B makes an angle β with the barrier. The coefficient of restitution between B and the barrier is $\frac{4}{5}$. As a result of the collision, the velocity of B is reduced to $\frac{12}{25}\sqrt{5}\,u$. Find the value of $\sin(\alpha + \beta)$. **[6]**

Solution 6

(a)

Mark scheme

- KE condition: $\frac{9}{2} \cdot \frac{1}{2}(2m)v_A^2 = \frac{1}{2}mv_B^2$, so $v_B = 3v_A$. Conservation of momentum: $2mu = 2mv_A + mv_B$, i.e. $2u = 2v_A + v_B$. With $v_B = 3v_A$: $v_A = \frac{2}{5}u$, $v_B = \frac{6}{5}u$ (AG).

Solution 6

(b) Mark scheme

- Let $w = \frac{12}{25}\sqrt{5}\,u$ be the speed after the collision. Parallel to wall (unchanged): $v_B \cos\alpha = w\cos\beta$, i.e. $\sqrt{5}\cos\alpha = 2\cos\beta$. Perpendicular to wall (restitution): $e\,v_B \sin\alpha = w\sin\beta$, i.e. $2\sin\alpha = \sqrt{5}\sin\beta$. Squaring and using $\sin^2 + \cos^2 = 1$: $\frac{576}{625}\sin^2\alpha + \frac{36}{25}(1 - \sin^2\alpha) = \frac{144}{125}$, giving $\sin^2\alpha = \frac{5}{9}$, so $\sin\alpha = \frac{1}{3}\sqrt{5}$, $\cos\alpha = \frac{2}{3}$, $\sin\beta = \frac{2}{3}$, $\cos\beta = \frac{1}{3}\sqrt{5}$. Hence
  $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{5}{9} + \frac{4}{9} = 1.$

Question 6

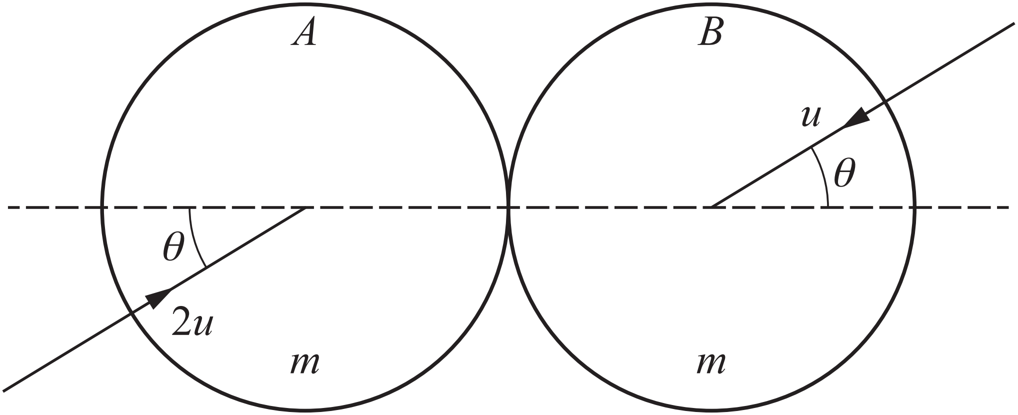
9231/33 J25 ▪ Q6 ▪ 8 marks

Two identical uniform smooth spheres A and B, each with mass m, are moving on a horizontal surface with speeds $2u$ and u respectively when they collide. Immediately before the collision, the spheres are moving parallel to each other in opposite directions such that their directions of motion each make an angle θ with the line of centres (see diagram). As a result of the collision, B moves in a direction which is perpendicular to its initial direction of motion. The coefficient of restitution between the spheres is e.

(a) Find an expression for $\tan\theta$ in terms of e. **[6]**

(b) As a result of the collision, A moves in a direction which is perpendicular to the line of centres. Find the value of θ. **[2]**

Question 6



Solution 6

(a)

Mark scheme

- Let v, w be the components of A, B along the line of centres after the collision. Momentum: $2mu\cos\theta - mu\cos\theta = mv + mw$. Restitution: $w - v = e(3u\cos\theta)$. Solving: $w = \frac{1}{2}(3e+1)u\cos\theta$. As B moves perpendicular to its initial direction, $\dfrac{\frac{1}{2}(3e+1)u\cos\theta}{u\sin\theta} = \tan\theta$, giving $\tan^2\theta = \frac{1}{2}(3e+1)$, so $\tan\theta = \sqrt{\frac{1}{2}(3e+1)}$.

Solution 6

(b)

Mark scheme

- A moves perpendicular to the line of centres, so $v = 0$. With $v = \frac{1}{2}(1 - 3e)u\cos\theta = 0$, $e = \frac{1}{3}$. Then $\tan^2\theta = \frac{1}{2}(3 \cdot \frac{1}{3} + 1) = 1$, so $\tan\theta = 1$ and $\theta = 45$.