Past-paper walk-through

Linear motion under a variable force ▪ 3.5

eXercise

Questions in this set

- 9231/31 N25 Q3 ▪ 7 marks
- 9231/32 N25 Q7 ▪ 10 marks
- 9231/34 N25 Q5 ▪ 9 marks
- 9231/31 J25 Q1 ▪ 5 marks
- 9231/33 J25 Q3 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

9231/31 N25 ▪ Q3 ▪ 7 marks

A particle P is moving in a straight horizontal line. At time t s, the displacement of P from a fixed point O on the line is x m and the velocity of P is v m s^{-1}. The acceleration of P is $\frac{1}{2}(v^2 + 4)$ m s^{-2} in the direction PO. Initially P is at O and is moving with velocity 2 m s^{-1}.

(a) Find an expression for x in terms of t. **[5]**

(b) Find the time when P next goes through O. **[2]**

Solution 3

(a)

Mark scheme

- $\dfrac{dv}{dt} = -\frac{1}{2}(v^2 + 4)$ (direction PO). Separating: $\frac{1}{2}\tan^{-1}\left(\frac{1}{2}v\right) = -\frac{1}{2}t + A$. At $t = 0$, $v = 2$: $A = \frac{1}{2}\tan^{-1}(1) = \frac{1}{8}\pi$. So $v = \dfrac{dx}{dt} = 2\tan\left(\frac{1}{4}\pi - t\right)$, giving $x = 2\ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + B$. At $t = 0$, $x = 0$: $B = \ln 2$. So $x = 2\ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2$ $\left[= \ln\left(2\cos^2\left(t - \frac{1}{4}\pi\right)\right)\right]$.

Solution 3

(b)

Mark scheme

- P next at O when $2\ln\left(\cos\left(t-\frac{1}{4}\pi\right)\right)+\ln 2=0 \Rightarrow 2\cos^2\left(t-\frac{1}{4}\pi\right)=1 \Rightarrow t=\frac{1}{2}\pi.$

Question 7

9231/32 N25 ▪ Q7 ▪ 10 marks

A particle P of mass m kg moving along a rough horizontal table has displacement x m from a fixed point O on the table and velocity v m s^{-1} at time t s. The particle P is subject to a resistive force of magnitude $mgkv$ N, where k is a positive constant, and a frictional force of magnitude μmg. The particle P is initially at O with speed U m s^{-1}.

(a) Show that $t = \dfrac{1}{gk}\ln\left(\dfrac{kU+\mu}{kv+\mu}\right)$. **[4]**

(b) It is given that $U = 10$, $k = 0.04$ and $\mu = 0.2$. Find the distance P moves before coming to rest. **[4]**

(c) Find the average speed of P over the period it is moving. **[2]**

Solution 7

(a) Mark scheme

- $m\dfrac{dv}{dt} = -(mgkv + \mu mg) = -mg(kv + \mu)$, so
$\displaystyle\int_U^v \frac{dv}{kv+\mu} = -\int_0^t g\,dt \Rightarrow \tfrac{1}{k}\ln\frac{kv+\mu}{kU+\mu} = -gt \Rightarrow t = \frac{1}{gk}\ln\left(\frac{kU+\mu}{kv+\mu}\right)$. AG.

Solution 7

(b)

Mark scheme

- Using $v\dfrac{dv}{dx} = -g(kv+\mu)$: $x = \dfrac{1}{g}\left[\dfrac{U}{k} - \dfrac{\mu}{k^2}\ln\dfrac{kU+\mu}{\mu}\right]$. With $U = 10$, $k = 0.04$, $\mu = 0.2$, $g = 10$: $x = \frac{1}{10}[250 - 125\ln 3] \approx 11.3$ m.

Solution 7

(c)

Mark scheme

- Time to stop $t = \dfrac{1}{gk} \ln \dfrac{kU + \mu}{\mu} = \frac{1}{0.4} \ln 3 = 2.747$ s. Average speed $= \dfrac{11.27}{2.747} \approx 4.10$ m s^{-1}.

Question 5

9231/34 N25 ▪ Q5 ▪ 9 marks

A particle P of mass m kg is projected vertically upwards from a point O with an initial speed of 20 m s^{-1} and moves under gravity. There is a resistive force of magnitude $0.025mv^2$ N, where v m s^{-1} is the speed of P at time t s after projection. The displacement of P from O is x m at time t s after projection.

(a) Find an expression for v in terms of x, while P is moving upwards. **[5]**

(b) Find an expression for v in terms of t, while P is moving upwards. **[4]**

Solution 5

(a) Mark scheme

- Moving up: $mv\frac{dv}{dx} = -0.025v^2m - 10m$, so $\int \frac{v}{10 + 0.025v^2}\,dv = -\int dx$, giving $\frac{1}{2}\ln(v^2 + 400) = -0.025x + c$. At $x = 0$, $v = 20$: $c = \frac{1}{2}\ln 800$. So $v^2 + 400 = 800\mathrm{e}^{-0.05x}$, i.e. $v = 20\sqrt{2\mathrm{e}^{-\frac{1}{20}x} - 1}$.

Solution 5

(b)

Mark scheme

- Moving up: $m\dfrac{dv}{dt} = -0.025v^2 m - 10m$, so $-\displaystyle\int dt = \int \frac{1}{10 + 0.025v^2}\, dv$, giving $-t + k = 2\tan^{-1}\left(\dfrac{v}{20}\right)$. At $t = 0$, $v = 20$: $k = 2\tan^{-1}(1) = \frac{1}{2}\pi$. So $v = 20\tan\left(\frac{1}{4}\pi - \frac{1}{2}t\right)$.

Question 1

9231/31 J25 ▪ Q1 ▪ 5 marks

A particle P of mass 8 kg is moving in a straight horizontal line. At time t s, P has displacement x m from a fixed point O on the line and velocity v m s^{-1}. The only horizontal force acting on P has magnitude $(x^3 + 4x)$ N and acts in the direction OP. When $t = 0$, $x = 0$ and $v = 1$.

(a) Find an expression for v in terms of x, giving your answer in the form $v = ax^2 + b$, where a and b are constants to be determined. **[3]**

(b) Find an expression for x in terms of t. **[2]**

Solution 1

(a)

Mark scheme

- N2L: $x^3 + 4x = 8v\dfrac{dv}{dx}$. Separating: $\displaystyle\int (x^3 + 4x)\,dx = \int 8v\,dv$, so $\frac{1}{4}x^4 + 2x^2 + c = 4v^2$. At $x = 0$, $v = 1$: $c = -4$. Hence $v^2 = \frac{1}{16}x^4 + \frac{1}{2}x^2 + 1 = \left(\frac{1}{4}x^2 + 1\right)^2$, so $v = \frac{1}{4}x^2 + 1$ ($a = \frac{1}{4}$, $b = 1$).

Solution 1

(b) Mark scheme

- $\dfrac{dx}{dt} = \frac{1}{4}x^2 + 1$, so $\displaystyle\int \frac{1}{\frac{1}{4}x^2 + 1}\,dx = \int dt$, giving
  $t = 4\int \dfrac{1}{x^2+4}\,dx = 2\tan^{-1}\left(\frac{1}{2}x\right) + C$. At $t = 0$, $x = 0$: $C = 0$. So $x = 2\tan\left(\frac{1}{2}t\right)$.

Question 3

9231/33 J25 ▪ Q3 ▪ 8 marks

A ball of mass m kg is projected vertically upwards with initial speed U m s^{-1} and moves under gravity. At time t s after projection, the ball has travelled a distance x m and its speed is v m s^{-1}. There is a resistive force of magnitude mkv^2 N, where k is a positive constant.

(a) Show that the distance travelled by the ball when it is moving upwards is

$$x = \frac{1}{2k}\ln\left(\frac{g + kU^2}{g + kv^2}\right).$$ **[4]**

(b) It is given that $k = 0.025$ and that $U = 20$. Find the time taken for the ball to reach its maximum height. **[4]**

Solution 3

(a) Mark scheme

- Moving up: $-mg - mkv^2 = mv\dfrac{dv}{dx}$, so $\displaystyle\int dx = -\int \frac{v}{g+kv^2}\,dv$, giving $x = -\dfrac{1}{2k}\ln(g+kv^2) + c$. At $x = 0$, $v = U$: $c = \dfrac{1}{2k}\ln(g+kU^2)$. Hence $x = \dfrac{1}{2k}\ln\left(\dfrac{g+kU^2}{g+kv^2}\right)$ (AG).

Solution 3

(b) Mark scheme

- Moving up: $10 + \frac{1}{40}v^2 = -\dfrac{dv}{dt}$, so $\displaystyle\int dt = -\int \frac{40}{400 + v^2}\, dv$, giving $t = -2\tan^{-1}\left(\frac{1}{20}v\right) + C$. At $t = 0$, $v = 20$: $C = \frac{1}{2}\pi$. At maximum height $v = 0$: $t = -2\tan^{-1}(0) + \frac{1}{2}\pi = 1.57$ s.