Exercise walk-through

Linear motion under a variable force ▪ 3.5

eXercise

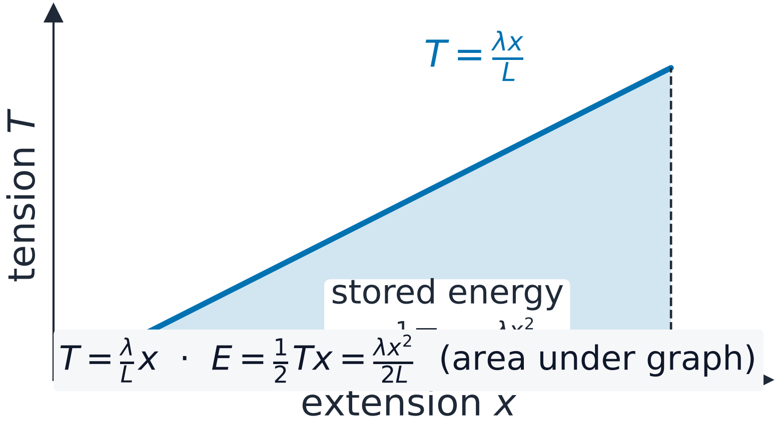
You must be able to

- choose $a = v\dfrac{dv}{dx}$ for a position-dependent force
- set up and solve the resulting differential equation
- use $a = \dfrac{dv}{dt}$ for a time-dependent force

Method — Force depending on position

8 **kg under force** $(x^3 + 4x)$ **N,** $v = 1$ **at** $x = 0$**:** $8v\dfrac{dv}{dx} = x^3 + 4x$; integrate $\to$ $4v^2 = \frac{1}{4}x^4 + 2x^2 + 4$, so $v = \frac{1}{4}x^2 + 1$.

Method — Force depending on position



$T = \frac{\lambda}{L}x \;\cdot\; E = \frac{1}{2}Tx = \frac{\lambda x^2}{2L}$ (area under graph)

A tension–extension line — a spring is one example of a variable force

Practice 2.1 ▪ 1 mark

State the form of acceleration to use when a force depends on **position** x.

Solution 2.1

Method: Force depending on position

Worked steps

$$a = v\frac{dv}{dx} \quad \boxed{\textbf{B1}}.$$

Practice 2.2 ▪ 2 marks

A particle has $v\frac{dv}{dx} = 2x$. Find v^2 in terms of x, given $v = 0$ at $x = 0$.

Solution 2.2

Worked steps

$\int v\,dv = \int 2x\,dx \Rightarrow \frac{1}{2}v^2 = x^2 + C$; $v = 0$ at $x = 0 \Rightarrow C = 0$, so $v^2 = 2x^2$ M1 A1.

Exam-style 3.1 ▪ 1 mark

For a force that depends on position, acceleration is best written as:

A $\dfrac{dv}{dt}$

B $v\dfrac{dv}{dx}$

C $\dfrac{dx}{dt}$

D $\dfrac{d^2x}{dt^2}$ only

Solution 3.1

Method: Force depending on position

A $\dfrac{dv}{dt}$

B $v\dfrac{dv}{dx}$ ✓

C $\dfrac{dx}{dt}$

D $\dfrac{d^2x}{dt^2}$ only

Worked steps

B. Use $a = v\frac{dv}{dx}$ for a position-dependent force.

Exam-style 3.2 ▪ 2 marks

A particle of mass 2 kg moves along a straight line under a force $F = 6x$ N in the direction of motion. At $x = 0$ the speed is 2 m s^{-1}.

(a) Form a differential equation in v and x.

(b) Find v when $x = 2$.

Solution 3.2

Method: Force depending on position

Worked steps

(a) $2v\dfrac{dv}{dx} = 6x$, i.e. $v\dfrac{dv}{dx} = 3x$ **M1** **A1**.

(b) $\int v\,dv = \int 3x\,dx \Rightarrow \frac{1}{2}v^2 = \frac{3}{2}x^2 + C$ **M1**; at $x = 0$, $v = 2 \Rightarrow C = 2$ **M1**; at $x = 2$: $\frac{1}{2}v^2 = 6 + 2 = 8 \Rightarrow v^2 = 16$ **M1**; $v = 4$ m s^{-1} **A1**.

Exam-style 3.3 ▪ 4 marks

A particle of mass 0.5 kg moves so that its acceleration is $\dfrac{dv}{dt} = 4 - 2t$. At $t = 0$ it is at rest. Find its velocity when $t = 3$ and comment.

Solution 3.3

Worked steps

$v = \int (4 - 2t)\,dt = 4t - t^2 + C$; $v = 0$ at $t = 0 \Rightarrow C = 0$ **M1** **M1**; at $t = 3$: $v = 12 - 9 = 3$ m s^{-1} **M1**; the force has reversed (since $4 - 2t < 0$ for $t > 2$), so the particle is slowing after $t = 2$ **A1**.