Past-paper walk-through

Hooke's law ▪ 3.4

eXercise

Questions in this set

- 9231/31 N25 Q4 ▪ 6 marks
- 9231/32 N25 Q2 ▪ 6 marks
- 9231/34 N25 Q6 ▪ 9 marks
- 9231/31 J25 Q5 ▪ 6 marks
- 9231/33 J25 Q2 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

9231/31 N25 ▪ Q4 ▪ 6 marks

One end of a light elastic string of natural length a and modulus of elasticity $5mg$ is attached to a fixed point O. Two particles, P and Q, of masses m and $4m$ respectively are attached to the other end of the string and they hang vertically in equilibrium. Particle Q is then detached from the string, hence releasing particle P from rest. Find, in terms of a, the length of the string when the speed of particle P is first equal to $\sqrt{\frac{7}{5}ag}$. **[6]**

Solution 4

- In equilibrium (mass $5m$): $\dfrac{5mge}{a} = 5mg$, so the extension $e = a$. After Q is detached, energy conservation for P (let x be the string length when its speed is $\sqrt{\frac{7}{5}ag}$): $\dfrac{5mga^2}{2a} - \dfrac{5mg(x-a)^2}{2a} = mg(e+a-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$. This gives $25x^2 - 60ax + 27a^2 = 0$, so $x = \frac{9}{5}a$ (only valid root).

Question 2

9231/32 N25 ▪ Q2 ▪ 6 marks

One end of a light elastic string of natural length a and modulus of elasticity $2mg$ is attached to a fixed point A on a rough horizontal surface. The other end is attached to a particle P of mass m. The particle and string rest on the surface. The coefficient of friction between P and the surface is μ. The particle P is initially held in equilibrium at a distance $\frac{4}{3}a$ from A, then released from rest.

(a) Given that the string never becomes slack, find the minimum value of μ. **[3]**

(b) It is now given that $\mu = \frac{1}{2}$. Find the extension of the string when the particle comes to rest. **[3]**

Solution 2

(a) Mark scheme

- Initial extension $\frac{a}{3}$. The string just reaches natural length with zero speed when EPE = friction work: $\frac{1}{2} \cdot \frac{2mg}{a}\left(\frac{a}{3}\right)^2 = \mu mg \cdot \frac{a}{3} \Rightarrow \frac{mga}{9} = \frac{\mu mga}{3} \Rightarrow \mu_{\min} = \frac{1}{3}$.

Solution 2

(b) Mark scheme

- Let the particle move a distance d. Energy: $\frac{mg}{a}\left[\left(\frac{a}{3}\right)^2 - \left(\frac{a}{3} - d\right)^2\right] = \frac{mg}{2}d \Rightarrow \frac{2}{3}d - \frac{d^2}{a} = \frac{d}{2} \Rightarrow d = \frac{a}{6}$. Extension at rest $= \frac{a}{3} - \frac{a}{6} = \frac{a}{6}$.

Question 6

9231/34 N25 ▪ Q6 ▪ 9 marks

A and B are two fixed points at a distance $22a$ apart, with B vertically below A. A light elastic string of natural length $4a$ and modulus of elasticity $5mg$ has one end attached to A and the other end attached to a particle P of mass km. Another light elastic string of natural length $8a$ and modulus of elasticity $6mg$ has one end attached to B and the other end attached to P. Particle P is vertically above B.

(a) Show that, when the system is in equilibrium, $BP = \dfrac{57a - 2ak}{4}$. **[5]**

(b) The particle P is pulled vertically upwards so that $BP = 18a$, and is then released from rest. In its subsequent motion, P first comes to instantaneous rest at the point where $BP = 8a$. Find the value of k. **[4]**

Solution 6

(a) Mark scheme

- N2L for P: $T_1 = T_2 + kmg$. Hooke's law: $T_1 = \dfrac{5mg(22a - BP - 4a)}{4a}$ (string AP, length $22a - BP$) and $T_2 = \dfrac{6mg(BP - 8a)}{8a}$ (string BP). Substituting: $\dfrac{5mg(18a - BP)}{4a} = \dfrac{6mg(BP - 8a)}{8a} + kmg$, giving $8BP = 114a - 4ak$, so $BP = \dfrac{57a - 2ak}{4}$ (AG).

Solution 6

(b)

Mark scheme

- Energy conservation (instantaneous rest at both ends, so no KE). At $BP = 18a$: $AP = 4a$ (natural length, 0 EPE), BP extension $10a$. At $BP = 8a$: $AP = 14a$ (extension $10a$), BP natural (0 EPE). P descends $10a$: $\dfrac{6mg(10a)^2}{2 \cdot 8a} + 18kmga = \dfrac{5mg(10a)^2}{2 \cdot 4a} + 8kmga$. Solving: $37.5mga + 18kmga = 62.5mga + 8kmga \Rightarrow 10kmga = 25mga$, so $k = \frac{5}{2}$.

Question 5

9231/31 J25 ▪ Q5 ▪ 6 marks

One end of a light elastic string of natural length 0.5 m and modulus of elasticity 14 N is attached to a fixed point *A* on a smooth plane. The plane makes an angle α to the horizontal, where $\tan\alpha = \frac{7}{24}$. A particle *P* of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane. The particle *P* is initially held on the plane above the level of *A*, where $AP = 0.8$ m. The particle *P* is then released from rest. Find the maximum velocity of *P* during the subsequent motion. **[6]**

Solution 5

Mark scheme

- At $v_{\max}$ (zero acceleration): $T = 2g\sin\alpha = \frac{14}{25}g$ ($\sin\alpha = \frac{7}{25}$). $\dfrac{14x}{0.5} = \frac{14}{25}g$ gives extension $x = \frac{1}{5}$. Energy from start ($AP = 0.8$, at rest, extension 0.3) to max-velocity point ($AP = 0.7$ below A, extension 0.2), a slope distance of 1.5 m: $2g\sin\alpha(0.8+0.5+0.2) + \dfrac{14(0.3)^2}{2(0.5)} = \dfrac{14(0.2)^2}{2(0.5)} + \frac{1}{2}(2)v^2$. So $v^2 = 0.91g = 9.1$, giving $v = 3.02$ m s^{-1}.

Question 2

9231/33 J25 ▪ Q2 ▪ 8 marks

A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity mg. The other end of the string is attached to a fixed point O on a rough plane inclined at an angle of 30 to the horizontal. The particle P is held at rest at point O before being released. The frictional force acting on P as it slides down the plane is $\frac{11}{30}mg$.

(a) Find, in terms of a, the distance that P moves down the plane before coming to rest. **[5]**

(b) It is given that P remains at rest in this new position. Find, in terms of m and g, the magnitude of the frictional force in this position. **[3]**

Solution 2

(a)

Mark scheme

- Energy equation (down a distance x, at rest): $\dfrac{mg}{2a}(x-a)^2 = mgx\sin 30 - \frac{11}{30}mgx$. Simplifying: $15x^2 - 34ax + 15a^2 = 0$, i.e. $(3x-5a)(5x-3a) = 0$. So $x = \frac{5}{3}a$ (rejecting $x = \frac{3}{5}a$).

Solution 2

(b)

Mark scheme

- At $x = \frac{5}{3}a$, tension $T = \dfrac{mg(x-a)}{a} = \dfrac{mg(\frac{2}{3}a)}{a} = \frac{2}{3}mg$. For equilibrium: $F = T - mg\sin 30 = \frac{2}{3}mg - \frac{1}{2}mg = \frac{1}{6}mg$.