Past-paper walk-through

Circular motion ▪ 3.3

eXercise

Questions in this set

- 9231/31 N25 Q2 ▪ 4 marks
- 9231/31 N25 Q6 ▪ 8 marks
- 9231/32 N25 Q1 ▪ 4 marks
- 9231/32 N25 Q6 ▪ 8 marks
- 9231/34 N25 Q1 ▪ 5 marks
- 9231/34 N25 Q4 ▪ 8 marks
- 9231/31 J25 Q3 ▪ 7 marks
- 9231/31 J25 Q7 ▪ 10 marks
- 9231/33 J25 Q1 ▪ 3 marks
- 9231/33 J25 Q5 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

9231/31 N25 ▪ Q2 ▪ 4 marks

A particle P of mass m is moving in a horizontal circle with angular speed ω_1 on the smooth inner surface of a hemispherical shell of radius r. The angle between the upward vertical and the normal reaction of the surface on P is θ_1, where $\tan\theta_1 = \frac{3}{4}$. When the angular speed is increased to ω_2, the angle between the upward vertical and the normal reaction of the surface on P becomes θ_2, where $\tan\theta_2 = \frac{4}{3}$. Find the ratio $\dfrac{\omega_1}{\omega_2}$. **[4]**

Solution 2

Mark scheme

- First case: $N_1 \cos\theta_1 = mg$, $N_1 \sin\theta_1 = m(r\sin\theta_1)\omega_1^2$, giving $\frac{5}{4}g = r\omega_1^2$ (using $\cos\theta_1 = \frac{4}{5}$). Second case similarly gives $\frac{5}{3}g = r\omega_2^2$ (using $\cos\theta_2 = \frac{3}{5}$).

  Dividing: $\dfrac{\frac{5}{4}g}{\frac{5}{3}g} = \dfrac{\omega_1^2}{\omega_2^2} = \frac{3}{4}$, so $\dfrac{\omega_1}{\omega_2} = \frac{1}{2}\sqrt{3}$.

Question 6

9231/31 N25 ▪ Q6 ▪ 8 marks

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. Initially P is held with the string taut and making an angle of 60 with the upward vertical through O. The particle P is projected perpendicular to the string in a downwards direction with speed $\sqrt{17ag}$. It then starts to move along a circular path in a vertical plane with centre O (see diagram). At the lowest point of its path, vertically below O, the particle P collides with a stationary particle Q.

(a) Find, in terms of a and g, an expression for the speed of P immediately before the collision with Q. **[2]**

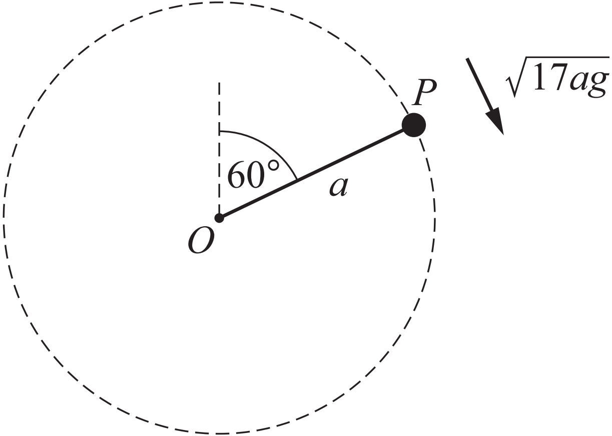
(b) As a result of the collision, P rebounds and moves back along a circular path with centre O. The string becomes slack when P reaches the point on the circle vertically

Question 6

above O. Find, in terms of a and g, an expression for the speed of P immediately after the collision with Q. **[3]**

(c) The mass of particle Q is km and the collision between P and Q is perfectly elastic. Find the value of k. **[3]**

Question 6



Solution 6

(a)

Mark scheme

- Energy conservation from the start (height $a\cos 60 = \frac{1}{2}a$ above O) to the lowest point (a below O), a drop of $\frac{3}{2}a$: $\frac{1}{2}m(17ag) + \frac{3}{2}mag = \frac{1}{2}mv^2$, so $v^2 = 20ag$ and $v = \sqrt{20ag}$.

Solution 6

(b)

Mark scheme

- At the top the string is slack, so $T = 0$ and N2L gives $\dfrac{mV^2}{a} = mg$, i.e. $V^2 = ag$. Energy from the lowest point (speed w_P) to the top (rise $2a$): $\frac{1}{2}mw_P^2 = \frac{1}{2}mV^2 + mg(2a)$, so $w_P^2 = ag + 4ag = 5ag$, giving $w_P = \sqrt{5ag}$.

Solution 6

(c)

Mark scheme

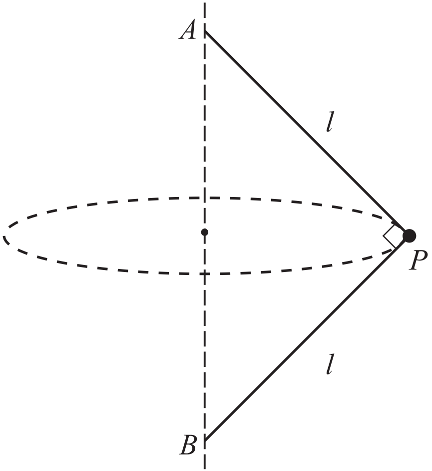
- Momentum: $mv = km\,w_Q - m\,w_P$ (P rebounds). Perfectly elastic ($e = 1$): $w_P + w_Q = v$. With $v = \sqrt{20ag} = 2\sqrt{5ag}$ and $w_P = \sqrt{5ag}$: $w_Q = v - w_P = \sqrt{5ag}$, then $2\sqrt{5ag} = k\sqrt{5ag} - \sqrt{5ag}$, so $k = 3$.

Question 1

9231/32 N25 ▪ Q1 ▪ 4 marks

A particle P of mass m is attached to two light inextensible strings each of length l. The end of one string is attached to a fixed point A and the end of the other to a fixed point B, with A vertically above B. Angle APB is a right angle. The particle P rotates in a horizontal circle at constant angular speed ω with both strings taut (see diagram). Find the tension in string AP in terms of m, g, l and ω. **[4]**

Question 1



Solution 1

Mark scheme

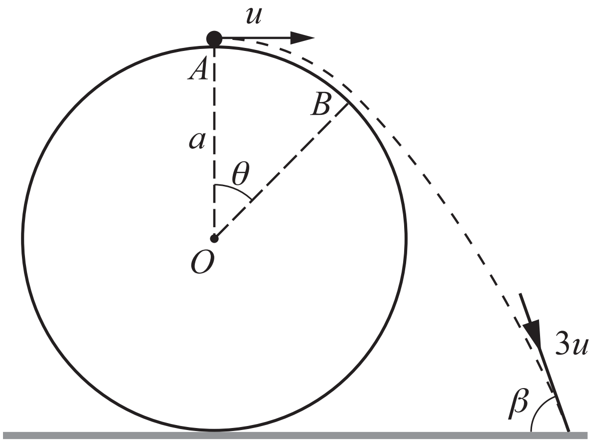
- Geometry gives each string at 45 to the vertical and circle radius $\frac{l}{\sqrt{2}}$. Vertical: $\frac{1}{\sqrt{2}}(T_A - T_B) = mg$; horizontal: $\frac{1}{\sqrt{2}}(T_A + T_B) = m\frac{l}{\sqrt{2}}\omega^2$, i.e. $T_A + T_B = ml\omega^2$ and $T_A - T_B = \sqrt{2}\,mg$. Hence $T_A = \frac{1}{2}m\big(l\omega^2 + \sqrt{2}\,g\big)$.

Question 6

9231/32 N25 ▪ Q6 ▪ 8 marks

A fixed smooth sphere with radius a and centre O rests on horizontal ground. A particle is projected horizontally from the highest point, A, of the sphere with speed u. The particle begins to move in a vertical circle along the surface of the sphere. The particle loses contact with the sphere at the point B, where the angle AOB is θ. After leaving the surface, the particle moves freely under gravity before striking the horizontal ground with speed $3u$ at an angle β to the horizontal (see diagram). Find the value of β. **[8]**

Question 6



Solution 6

Mark scheme

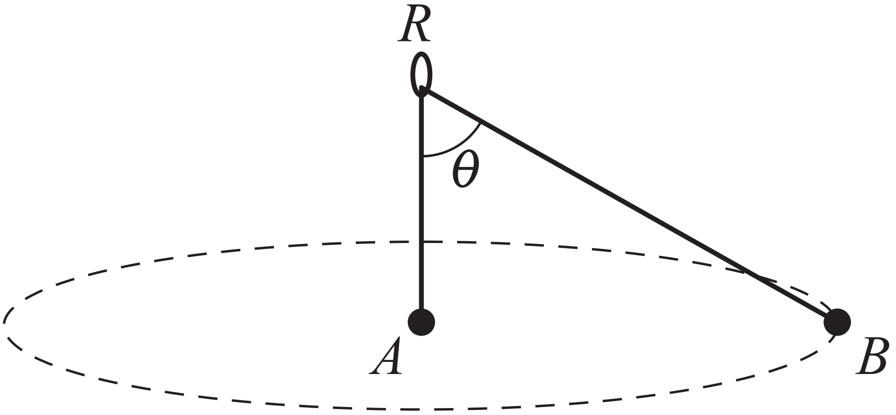
- On the sphere: $w^2 = u^2 + 2ga(1 - \cos\theta)$ and loss of contact $w^2 = ga\cos\theta$, giving $\cos\theta = \frac{u^2+2ga}{3ga}$. Energy from B to the ground: $9u^2 = w^2 + 2ga(1+\cos\theta) = 3ga\cos\theta + 2ga$, and with $3ga\cos\theta = u^2 + 2ga$ this gives $ga = 2u^2$, so $\cos\theta = \frac{5}{6}$. Conserving horizontal velocity, $3u\cos\beta = w\cos\theta = \frac{5}{6}u\sqrt{\frac{5}{3}}$, so $\cos\beta = \frac{5\sqrt{15}}{54}$ and $\beta = 69.0$.

Question 1

9231/34 N25 ▪ Q1 ▪ 5 marks

A light inextensible string of length $12a$ is threaded through a fixed smooth ring R. One end of the string is attached to a particle A of mass m. The other end of the string is attached to a particle B of mass $0.5m$. Particle A hangs in equilibrium vertically below the ring. Particle B moves with constant angular speed ω in a horizontal circle with particle A at its centre. The angle between AR and BR is θ (see diagram). Express ω in terms of g and a. **[5]**

Question 1



Solution 1

Mark scheme

- For A: $T = mg$. For B vertically: $T\cos\theta = 0.5mg$, so $\cos\theta = \frac{1}{2}$, $\theta = 60$. For B horizontally: $T\sin\theta = 0.5m(BR\sin\theta)\omega^2$. Geometry: $BR\cos\theta = AR$ with $AR + BR = 12a$ gives $BR = 8a$. Then $mg = 0.5m(8a)\omega^2 = 4ma\omega^2$, so $\omega = \frac{1}{2}\sqrt{\dfrac{g}{a}}$.

Question 4

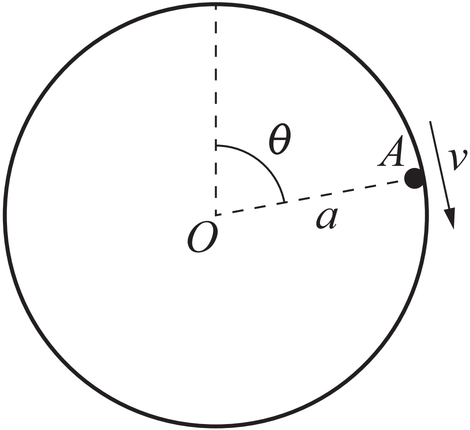
9231/34 N25 ▪ Q4 ▪ 8 marks

A fixed smooth spherical shell has centre O and radius a. A particle of mass m moves in complete vertical circles on the smooth inner surface of the shell, where the plane of the circular motion is vertical and passes through O. The particle has speed v when it is at point A, where OA makes an angle θ with the upward vertical through O, and $\cos\theta = \frac{1}{18}$ (see diagram).

(a) Show that $v \geqslant \frac{1}{3}\sqrt{26ag}$. **[5]**

(b) It is given that $v = \frac{1}{3}\sqrt{26ag}$. Find, in terms of m and g, an expression for the greatest possible value of the normal reaction between the shell and the particle. **[3]**

Question 4



Solution 4

(a)

Mark scheme

- At the top, $R \geqslant 0$ gives $\dfrac{mw^2}{a} \geqslant mg$, i.e. $w^2 \geqslant ag$. Energy (A to top, rise $a(1-\cos\theta)$): $\frac{1}{2}mv^2 - \frac{1}{2}mw^2 \geqslant mga(1-\cos\theta)$. Substituting $w^2 = ag$ and $\cos\theta = \frac{1}{18}$: $\frac{1}{2}mv^2 \geqslant \frac{13}{9}mga$, so $v \geqslant \frac{1}{3}\sqrt{26ag}$ (AG).

Solution 4

(b)

Mark scheme

- Greatest reaction is at the bottom. Energy (A to bottom, drop $a(1+\cos\theta)$): $\frac{1}{2}mv^2 + mga(1+\cos\theta) = \frac{1}{2}mV^2$, giving $V^2 = 5ag$. At the bottom,
  $R_{\max} - mg = \dfrac{mV^2}{a} = 5mg$, so $R_{\max} = 6mg$.

Question 3

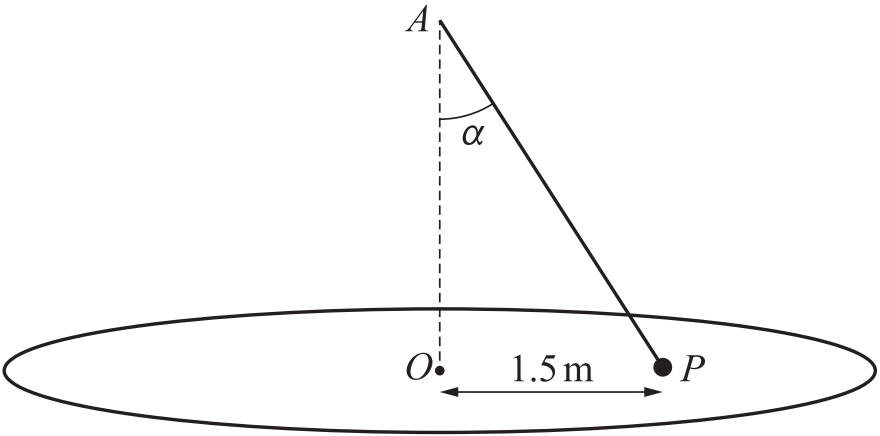
9231/31 J25 ▪ Q3 ▪ 7 marks

A rough horizontal disc, centre O, rotates with constant angular speed ω rad s^{-1}. A particle P of mass 1.6 kg lies on the disc at a distance 1.5 m from O, and is attached to a point A vertically above O by a light elastic string. The string has natural length 2 m, modulus of elasticity 32 N and makes an angle α with the vertical OA (see diagram). Particle P moves in a horizontal circle also at a constant angular speed ω rad s^{-1}. Particle P is on the point of slipping in the direction OP. The coefficient of friction between the particle and the disc is 0.5.

(a) Given that the tension in the string is 8 N, show that $\sin\alpha = 0.6$. **[2]**

(b) Find the number of revolutions per minute made by the disc and the particle P. **[5]**

Question 3



Solution 3

(a)

Mark scheme

- $T = \frac{\lambda x}{l}$: $8 = \frac{32x}{2}$, so the extension $x = 0.5$ and $AP = 2.5$. Then $\sin\alpha = \frac{1.5}{2.5} = 0.6$ (AG).

Solution 3

(b)

Mark scheme

- Vertically (↑): $R = 1.6g - 8\cos\alpha$ $(\cos\alpha = 0.8)$. Horizontally (centripetal, friction inward as P is on the point of slipping outward): $1.6 \cdot 1.5\,\omega^2 = 8\sin\alpha + \mu R$. Substituting: $2.4\omega^2 = 8(0.6) + 0.5(1.6g - 8(0.8)) = 9.6$, so $\omega^2 = 4$, $\omega = 2$. Revolutions per minute $= \dfrac{\omega}{2\pi} \times 60 = \dfrac{60}{\pi} = 19.1$.

Question 7

9231/31 J25 ▪ Q7 ▪ 10 marks

A fixed hollow sphere has radius a and centre O. The points A, B and C lie on the inner surface of the sphere with OA and BC horizontal. A portion of the sphere has been removed by a horizontal cut through points B and C at a vertical distance ka above the centre of the sphere, where k is a positive constant and $k < 1$. The points O, A, B and C all lie in the same vertical plane. OB makes an angle θ with the upward vertical through O (see diagram). A particle P of mass m is free to move on the smooth inner surface of the sphere. The particle P is projected vertically downwards from A with speed u and begins to move in a vertical circle.

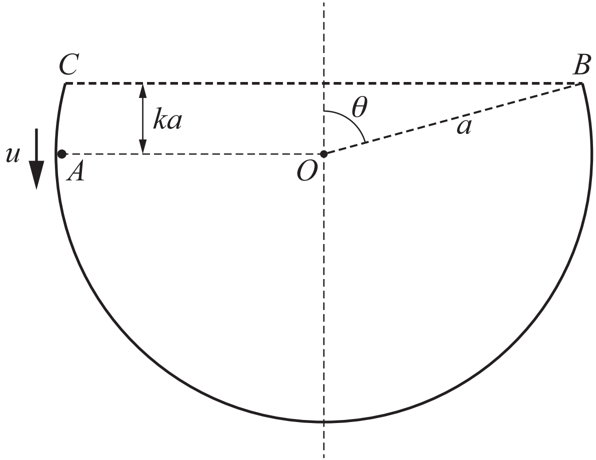
(a) In the case where $u = \sqrt{\frac{6}{5}ga}$, the reaction on P at B is half the reaction on P at A. Find the value of k. **[5]**

Question 7

(b) Find an expression for u, in terms of a and g, in the case that the particle just reaches B. **[1]**

(c) Find an expression for u, in terms of a and g, in the case that the particle passes through B and in its subsequent motion reaches C. **[4]**

Question 7



Solution 7

(a) Mark scheme

- At A (centripetal horizontal): $R_A = \dfrac{mu^2}{a}$. At B: $R_B = \dfrac{mv^2}{a} - mg\cos\theta$, with $\cos\theta = k$. Energy A to B (rise ka): $v^2 = u^2 - 2agk$. Using $R_B = \frac{1}{2}R_A$: $\dfrac{mv^2}{a} - mgk = \frac{1}{2}\dfrac{mu^2}{a}$, so $v^2 = \frac{1}{2}u^2 + agk$. Equating: $\frac{1}{2}u^2 + agk = u^2 - 2agk \Rightarrow u^2 = 6agk$. With $u^2 = \frac{6}{5}ga$: $k = \frac{1}{5}$.

Solution 7

(b)

Mark scheme

- Just reaches B means $R_B = 0$: $\dfrac{mv^2}{a} = mgk \Rightarrow v^2 = agk$, and $u^2 = v^2 + 2agk = 3agk = \frac{3}{5}ag$ (with $k = \frac{1}{5}$). So $u = \sqrt{\frac{3}{5}ag}$.

Solution 7

(c) Mark scheme

- At B the particle leaves the surface and travels as a projectile to C (same height, horizontal separation $2a\sin\theta$). Time of flight $t = \dfrac{2v\sin\theta}{g}$; horizontally $2a\sin\theta = v\cos\theta \cdot t = \dfrac{2v^2\sin\theta\cos\theta}{g}$, so $v^2 = \dfrac{ag}{\cos\theta} = 5ag$ (with $\cos\theta = k = \frac{1}{5}$). Then $u^2 = v^2 + 2agk = 5ag + \frac{2}{5}ag = \frac{27}{5}ag$, so $u = \sqrt{\frac{27}{5}ag}$.

Question 1

9231/33 J25 ▪ Q1 ▪ 3 marks

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle moves in a horizontal circle with constant angular speed ω and with the string inclined at an angle of θ to the downward vertical. Given that $\tan\theta = \frac{4}{3}$, find ω in terms of a and g. **[3]**

Solution 1

Mark scheme

- $T\cos\theta = mg$ and $T\sin\theta = m\omega^2 r$, with $r = a\sin\theta$. So $T = \frac{5}{3}mg$ $(\cos\theta = \frac{3}{5})$, and $\frac{5}{3}mg\sin\theta = m\omega^2 a\sin\theta$, giving $\omega^2 = \dfrac{5g}{3a}$, i.e. $\omega = \sqrt{\dfrac{5g}{3a}}$.

Question 5

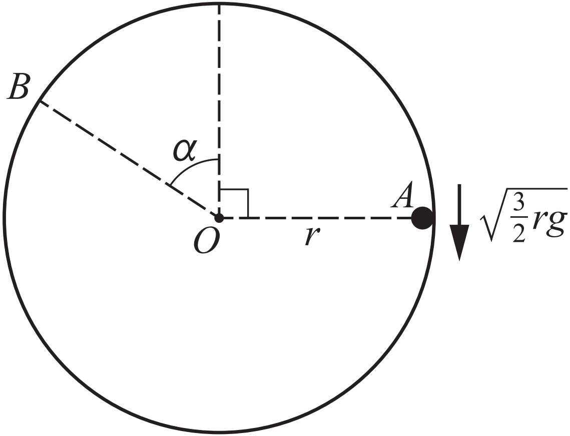
9231/33 J25 ▪ Q5 ▪ 8 marks

A hollow cylinder of radius r is fixed with its axis horizontal. Points A, B and O are in the same vertical plane perpendicular to the axis of the cylinder, with A and B on the smooth inner surface and O on the axis. OA and OB make angles 90 and α respectively with the upward vertical through O, with A and B on opposite sides of the vertical. A particle of mass m is projected vertically downwards from point A with speed $\sqrt{\frac{3}{2}rg}$ and moves in a vertical circle inside the cylinder (see diagram). The particle loses contact with the cylinder at point B.

(a) Find the value of α. **[4]**

(b) In the subsequent motion find, in terms of r, the greatest height above O reached by the particle. **[4]**

Question 5



Solution 5

(a)

Mark scheme

- Energy A to B (rise $r\cos\alpha$): $\frac{1}{2}mv^2 + mgr\cos\alpha = \frac{1}{2}m\left(\frac{3}{2}gr\right)$, so $v^2 = \frac{3}{2}gr - 2gr\cos\alpha$. Loses contact at B ($R = 0$): $mg\cos\alpha = \dfrac{mv^2}{r}$. Combining: $g\cos\alpha = \frac{3}{2}g - 2g\cos\alpha$, so $\cos\alpha = \frac{1}{2}$, $\alpha = 60$.

Solution 5

(b)

Mark scheme

- At B, $v^2 = \frac{1}{2}gr$ (with $\alpha = 60$). As a projectile, the rise above B is h where $0 = (v\sin 60)^2 - 2gh$, giving $h = \frac{3}{16}r$. Greatest height above O $= \frac{3}{16}r + r\cos 60 = \frac{3}{16}r + \frac{1}{2}r = \frac{11}{16}r$.