Past-paper walk-through

Motion of a projectile ▪ 3.1

eXercise

Questions in this set

- 9231/31 N25 Q7 ▪ 12 marks
- 9231/32 N25 Q4 ▪ 7 marks
- 9231/34 N25 Q2 ▪ 6 marks
- 9231/31 J25 Q2 ▪ 6 marks
- 9231/33 J25 Q7 ▪ 11 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 7

9231/31 N25 ▪ Q7 ▪ 12 marks

A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is 25 m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A, the direction of motion of P makes an angle of 45 with the downward vertical through A.

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A. **[5]**

(b) At point A, the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of 45 to the horizontal. Find the speed of P immediately before it collides with the barrier. **[3]**

(c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. **[4]**

Solution 7

(a) Mark scheme

- $\dfrac{dy}{dx} = \tan\theta - \dfrac{gx}{u^2\cos^2\theta} = \frac{4}{3} - \dfrac{2x}{45}$ (with $u = 25$, $\tan\theta = \frac{4}{3}$, $g = 10$). At A the direction is 45 to the downward vertical, so $\dfrac{dy}{dx} = -1$:
  $-1 = \frac{4}{3} - \frac{2x}{45} \Rightarrow x_A = \frac{105}{2} = 52.5$. Then $y_A = x_A\tan\theta - \dfrac{gx_A^2}{2u^2\cos^2\theta} = \frac{35}{4} = 8.75$.
  So $A = \left(\frac{105}{2}, \frac{35}{4}\right)$.

Solution 7

(b)

Mark scheme

- Horizontal velocity unchanged: $v_x = u\cos\theta = 15$. At A the direction is at 45 below the horizontal, so $v_y = -v_x = -15$. Speed $= \sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 15^2} = 15\sqrt{2}$ m s^{-1}.

Solution 7

(c) Mark scheme

- The velocity $(15, -15)$ is perpendicular to the 45 barrier, so the rebound speed is $w = ev = \frac{1}{9}(15\sqrt{2}) = \frac{5}{3}\sqrt{2}$, directed at 45. Projecting from $A = \left(\frac{105}{2}, \frac{35}{4}\right)$ until it lands (y drops $\frac{35}{4}$): $\frac{35}{4} = \frac{5}{3}\sqrt{2} \cdot \frac{1}{\sqrt{2}}t + \frac{1}{2}gt^2 \Rightarrow t = \frac{3}{2}$, so horizontal distance $x = (w \cos 45)t = \frac{5}{3} \cdot \frac{3}{2} = \frac{5}{2}$ metres.

Question 4

9231/32 N25 ▪ Q4 ▪ 7 marks

A particle Q is initially positioned at a distance d vertically above a particle P. Particle P is projected with speed U at an angle α above the horizontal. At the same time, Q is projected at an angle β below the horizontal. Both particles move freely under gravity. The particles collide at time T after the projections.

(a) Show that $d = UT(\sin\alpha + \cos\alpha\tan\beta)$. **[4]**

(b) The particles collide when P is at its maximum height. Given that $\alpha = 30$ and $\beta = 60$, find d in terms of U and g. **[3]**

Solution 4

(a)

Mark scheme

- Matching horizontal displacements gives Q's speed $\frac{U\cos\alpha}{\cos\beta}$. Vertical: $U\sin\alpha\, T - \frac{1}{2}gT^2 = d - \frac{U\cos\alpha}{\cos\beta}\sin\beta\, T - \frac{1}{2}gT^2$, so $d = UT\sin\alpha + UT\cos\alpha\tan\beta = UT(\sin\alpha + \cos\alpha\tan\beta)$. AG.

Solution 4

(b) Mark scheme

- At maximum height $T = \dfrac{U \sin \alpha}{g} = \dfrac{U}{2g}$. Then $d = U \cdot \frac{U}{2g}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \sqrt{3}\right) = \frac{U^2}{2g} \cdot 2 = \dfrac{U^2}{g}$.

Question 2

9231/34 N25 ▪ Q2 ▪ 6 marks

A particle P is projected with speed u m s^{-1} at an angle θ, where $\tan\theta = 2$, above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a time t s are denoted by x m and y m respectively.

(a) Use the equation of the trajectory given in the list of formulae (MF19) to show that $y = 2x - \dfrac{25x^2}{u^2}$. **[1]**

(b) In the subsequent motion, P passes through the point with coordinates $(8, 12)$. The particle then hits a fixed vertical barrier 7 m high that is at a horizontal distance of D m from the point of projection. Find the set of possible values of D. **[5]**

Solution 2

(a)

Mark scheme

- Trajectory $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$. With $\tan\theta = 2$, $\cos^2\theta = \frac{1}{5}$, $g = 10$:
  $y = 2x - \dfrac{10x^2}{2u^2 \cdot \frac{1}{5}} = 2x - \dfrac{25x^2}{u^2}$ (AG).

Solution 2

(b)

Mark scheme

- Through $(8,12)$: $12 = 16 - \dfrac{25\cdot 64}{u^2} \Rightarrow u^2 = 400$ ($u = 20$). So $y = 2x - \dfrac{25x^2}{400}$. Range ($y=0$): $D = 0$ or 32. Height 7 ($y = 7$): $D = 4$ or 28. Since the barrier is struck after P passes $(8,12)$ (on the descending part with $y \leqslant 7$), $28 \leqslant D \leqslant 32$.

Question 2

9231/31 J25 ▪ Q2 ▪ 6 marks

A particle P is projected with speed 24 m s^{-1} at an angle θ above the horizontal from a point O on a horizontal plane, and moves freely under gravity. At a horizontal distance 35 m from O, there is a vertical wall of height 10 m which is perpendicular to the vertical plane of motion of P.

(a) Determine the two values of θ for which P just clears the wall. **[4]**

(b) Given that P clears the wall, find the minimum distance from point O where P can land. **[2]**

Solution 2

(a)

Mark scheme

- Using the trajectory equation with $(35, 10)$, $u = 24$, $g = 10$: $10 = 35t - \dfrac{g}{2 \cdot 24^2} \cdot 35^2(1 + t^2)$ where $t = \tan\theta$, giving $6125t^2 - 20160t + 11885 = 0$ (OE $1225t^2 - 4032t + 2377 = 0$). So $\tan\theta = 0.769$ $(\theta = 37.6)$ or $\tan\theta = 2.52$ $(\theta = 68.4)$.

Solution 2

(b)

Mark scheme

- Using the trajectory with $y = 0$ and the larger value of θ (68.4): $0 = x\tan\theta - \frac{g}{2\cdot 24^2}x^2(1+\tan^2\theta)$, giving range $x = 39.5$ m (accept $39.4 \leqslant x \leqslant 39.5$). This is the minimum landing distance.

Question 7

9231/33 J25 ▪ Q7 ▪ 11 marks

A particle P is projected from a point O with speed U at an angle 45 above the horizontal and moves freely under gravity.

(a) State the vertical and horizontal components of velocity at time t. **[1]**

(b) At time T, particle P is moving at an angle of 60 below the horizontal. Show that $T = \dfrac{U}{2g}(\sqrt{2} + \sqrt{6})$. **[3]**

(c) At time T, the particle strikes a smooth horizontal plane at a point which is a horizontal distance D from O and a vertical distance H below O. Find the ratio $H : D$. **[4]**

(d) After striking the horizontal plane, P rebounds with speed w. The coefficient of restitution between P and the plane is $\frac{2}{3}$. Find w in terms of U. **[3]**

Solution 7

(a)

Mark scheme

- $v_y = U\sin 45 - gt$ and $v_x = U\cos 45$.

Solution 7

(b) Mark scheme

- At T: $\dfrac{v_y}{v_x} = -\tan 60$, so $\dfrac{\frac{U}{\sqrt{2}} - gT}{\frac{U}{\sqrt{2}}} = -\sqrt{3}$, giving $U - \sqrt{2}\,gT = -\sqrt{3}\,U$, i.e. $T = \dfrac{U}{\sqrt{2}\,g}(1+\sqrt{3}) = \dfrac{U}{2g}(\sqrt{2}+\sqrt{6})$ (AG).

Solution 7

(c)

Mark scheme

- $D = \dfrac{U}{\sqrt{2}} T$ and $H = \left| \dfrac{U}{\sqrt{2}} T - \tfrac{1}{2} g T^2 \right|$. Forming the ratio (substituting T): $\dfrac{H}{D} = \left| \dfrac{1-\sqrt{3}}{2} \right|$, so $H : D = 1 : 1 + \sqrt{3}$ (equivalently $\sqrt{3} - 1 : 2$).

Solution 7

(d) Mark scheme

- On the bounce the horizontal component is unchanged ($\frac{U}{\sqrt{2}}$) and the vertical component is reversed and scaled by $e = \frac{2}{3}$: $w_y = \frac{2}{3} \cdot \frac{\sqrt{3}}{\sqrt{2}} U = \frac{\sqrt{2}}{\sqrt{3}} U$. So $w^2 = w_y^2 + \left(\frac{U}{\sqrt{2}}\right)^2 = \frac{2}{3} U^2 + \frac{1}{2} U^2 = \frac{7}{6} U^2$, giving $w = U\sqrt{\frac{7}{6}}$.