

Exercise walk-through

Vectors ■ 1.6

eXercise

You must be able to

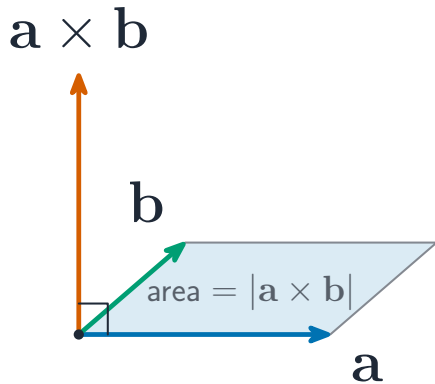
- compute $\mathbf{a} \times \mathbf{b}$ and use it for a normal/area
- write a plane as $\mathbf{r} \cdot \mathbf{n} = p$ or $ax + by + cz = d$
- find the angle between, and distance from, lines and planes

Method — Vector product & plane

$$\mathbf{a} = \mathbf{i} + \mathbf{j}, \mathbf{b} = \mathbf{j} + \mathbf{k}: \mathbf{a} \times \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}.$$

$$\text{Plane through } A \text{ with normal } \mathbf{n}: \mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}.$$

Method — Vector product & plane



Two vectors spanning a parallelogram with their vector product standing up

Practice 2.1 ■ 3 marks

Find $\mathbf{a} \times \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{k}$.

Solution 2.1

Method: Vector product & plane

Worked steps

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \mathbf{i}(1) - \mathbf{j}(2) + \mathbf{k}(-1) = \mathbf{i} - 2\mathbf{j} - \mathbf{k} \quad \text{M1 M1 A1}.$$

Practice 2.2 ■ 2 marks

Write the plane through $(1, 2, 3)$ with normal $\mathbf{n} = (1, 1, 1)$ in the form $ax + by + cz = d$.

Solution 2.2

Method: Vector product & plane

Worked steps

$$x + y + z = 1 + 2 + 3 = 6 \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

$\mathbf{a} \times \mathbf{b}$ is a vector that is:

A parallel to $\mathbf{a}$

B perpendicular to both $\mathbf{a}$ and $\mathbf{b}$

C equal to $\mathbf{a} \cdot \mathbf{b}$

D of length $|\mathbf{a}||\mathbf{b}| \cos \theta$

Solution 3.1

Method: Vector product & plane

- A parallel to $\mathbf{a}$
- B perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ ✓
- C equal to $\mathbf{a} \cdot \mathbf{b}$
- D of length $|\mathbf{a}||\mathbf{b}| \cos \theta$

Worked steps

B. The vector product is perpendicular to both vectors.

Exam-style 3.2 ■ 4 marks

Points $A(2, 1, 0)$, $B(3, 3, 1)$ and $C(1, 2, 3)$ are given.

(a) Find $\vec{AB} \times \vec{AC}$.

(b) Hence find the equation of the plane through A , B , C .

Solution 3.2

Method: Vector product & plane

Worked steps

$$(a) \vec{AB} = (1, 2, 1), \vec{AC} = (-1, 1, 3) \quad \text{M1}; \text{ cross} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{vmatrix} = \mathbf{i}(6 - 1) - \mathbf{j}(3 + 1) + \mathbf{k}(1 + 2) = (5, -4, 3) \quad \text{M1 M1 A1}.$$

$$(b) \text{ normal } (5, -4, 3) \text{ through } A(2, 1, 0): 5x - 4y + 3z = 6 \quad \text{M1 M1 A1}.$$

Exam-style 3.3 ■ 4 marks

Find the acute angle between the planes $x + 2y + 2z = 5$ and $2x - y + 2z = 1$.

Solution 3.3

Worked steps

normals $(1, 2, 2)$ and $(2, -1, 2)$; dot $= 2 - 2 + 4 = 4$ **M1**; $|\mathbf{n}_1| = 3$, $|\mathbf{n}_2| = 3$ **M1**;
 $\cos \theta = \frac{4}{9} \Rightarrow \theta = 63.6^\circ$ **M1** **A1**.