Exercise walk-through

Polar coordinates ▪ 1.5

eXercise

You must be able to

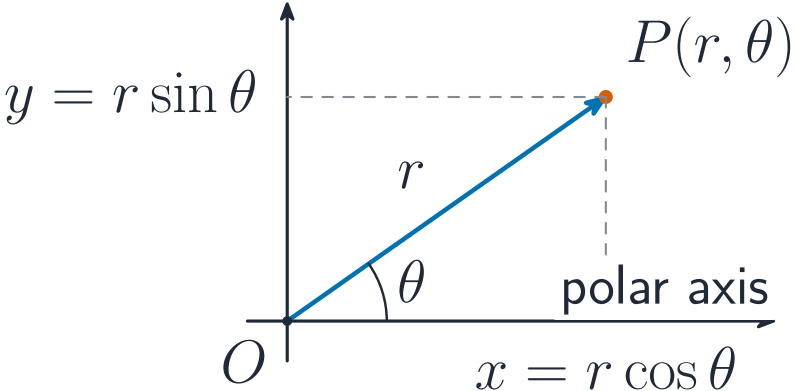
- convert between (r, θ) and (x, y)
- sketch a simple polar curve (e.g. a cardioid)
- find an area with $\frac{1}{2}\int r^2\, d\theta$

Method — Conversion & area

Convert $r = 4\cos\theta$**:** $\times r \to r^2 = 4r\cos\theta$, so $x^2 + y^2 = 4x$, i.e. $(x-2)^2 + y^2 = 4$ (circle).

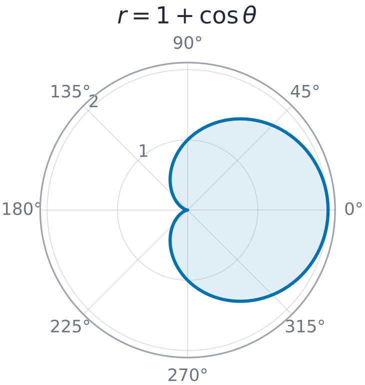
Area: $A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta$.

Method — Conversion & area



A point reached by distance r at angle theta, with x and y components

Method — Conversion & area



cardioid · polar area $A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$

Polar curves are traced as the angle varies

Practice 2.1 ▪ 2 marks

Convert the point $(r, \theta) = \left(2, \frac{\pi}{3}\right)$ to Cartesian coordinates.

Solution 2.1

Method: Conversion & area

Worked steps

$x = 2\cos\frac{\pi}{3} = 1,\ y = 2\sin\frac{\pi}{3} = \sqrt{3}$ M1 A1.

Practice 2.2 ▪ 1 mark

Write down the formula for the area of a polar sector.

Solution 2.2

Method: Conversion & area

Worked steps

$A = \frac{1}{2} \int r^2 \, d\theta$ **B1**.

Exam-style 3.1 ▪ 1 mark

The polar equation $r = 2\sin\theta$ represents:

A a line

B a circle

C a parabola

D a spiral

Solution 3.1

Method: Conversion & area

A a line

B a circle ✓

C a parabola

D a spiral

Worked steps

B. $r = 2\sin\theta \Rightarrow r^2 = 2r\sin\theta \Rightarrow x^2 + y^2 = 2y$, a circle.

Exam-style 3.2 ▪ 3 marks

A curve has polar equation $r = 2(1 + \cos\theta)$ (a cardioid).
(a) Find r when $\theta = 0$ and $\theta = \frac{\pi}{2}$, and state the value of θ at which $r = 0$.
(b) Find the area enclosed by the curve, using $A = \frac{1}{2}\int_0^{2\pi} 4(1 + \cos\theta)^2 \, d\theta$.

Solution 3.2

Method: Conversion & area

Worked steps

(a) $\theta = 0$: $r = 4$; $\theta = \frac{\pi}{2}$: $r = 2$ **B1** **B1**; $r = 0$ when $\cos\theta = -1$, i.e. $\theta = \pi$ **B1**.
(b) $4(1 + \cos\theta)^2 = 4 + 8\cos\theta + 4\cos^2\theta = 6 + 8\cos\theta + 2\cos 2\theta$ **M1**; $A = \frac{1}{2}\left[6\theta + 8\sin\theta + \sin 2\theta\right]_0^{2\pi}$ **M1** **M1**; the sine terms vanish at both limits, so $A = \frac{1}{2}(6 \cdot 2\pi) = 6\pi$ **M1** **A1**.

Exam-style 3.3 ▪ 4 marks

Convert $r = \dfrac{2}{1+\cos\theta}$ to Cartesian form.

Solution 3.3

Method: Conversion & area

Worked steps

$r + r\cos\theta = 2 \Rightarrow r = 2 - x$ (since $r\cos\theta = x$) **M1**; $r^2 = (2-x)^2 \Rightarrow x^2 + y^2 = 4 - 4x + x^2$ **M1** **M1**; $y^2 = 4 - 4x$, i.e. $y^2 = -4(x-1)$ **A1**.