

Exercise walk-through

Matrices ■ 1.4

eXercise

You must be able to

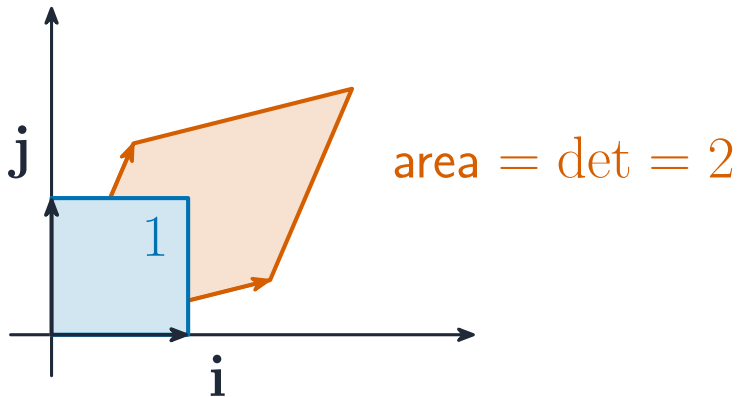
- multiply matrices and use I
- find $\det A$ and A^{-1} for a 2×2 matrix; use $(AB)^{-1} = B^{-1}A^{-1}$
- identify the transformation a matrix represents (and its area scale factor)

Method — Inverse & transformation

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}: \det A = 12 - 2 = 10; A^{-1} = \frac{1}{10} \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix}.$$

Transformations: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is a 90° rotation; $\det = 1$ (area preserved).

Method — Inverse & transformation



A unit square mapped to a parallelogram by a matrix

Practice 2.1 ■ 1 mark

Find $\det \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix}$.

Solution 2.1

Method: Inverse & transformation

Worked steps

$$\det = 5(4) - 2(3) = 14 \text{ (B1)}.$$

Practice 2.2 ■ 2 marks

Find the inverse of $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$.

Solution 2.2

Method: Inverse & transformation

Worked steps

$$\det = 2 - 1 = 1; \text{ inverse} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \text{ M1 A1}.$$

Exam-style 3.1 ■ 1 mark

The matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ represents:

- A** a rotation
- B** an enlargement, scale factor 2
- C** a reflection
- D** a shear

Solution 3.1

Method: Inverse & transformation

- A a rotation
- B an enlargement, scale factor 2**
- C a reflection
- D a shear



Worked steps

B. A scalar multiple of I is an enlargement; here scale factor 2.

Exam-style 3.2 ■ 2 marks

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}.$$

- (a) Find AB .
- (b) Find $\det(AB)$ and verify it equals $\det A \times \det B$.

Solution 3.2

Method: Inverse & transformation

Worked steps

$$(a) AB = \begin{pmatrix} 1 \cdot 2 + 2 \cdot 1 & 1 \cdot 0 + 2 \cdot 3 \\ 3 \cdot 2 + 4 \cdot 1 & 3 \cdot 0 + 4 \cdot 3 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 10 & 12 \end{pmatrix} \quad \text{M1 A1}.$$

$$(b) \det(AB) = 48 - 60 = -12; \det A = -2, \det B = 6, \text{ product} = -12$$

M1 M1 A1.

Exam-style 3.3 ■ 1 mark

The transformation T has matrix $M = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$.

- (a) Find the area scale factor of T .
- (b) Find M^{-1} and hence the image-to-object matrix that undoes T .

Solution 3.3

Method: Inverse & transformation

Worked steps

(a) area scale factor = $\det M = 6$ **B1**.

(b) $M^{-1} = \frac{1}{6} \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ **M1** **A1**; this matrix undoes T **A1**.