Past-paper walk-through

Summation of series ▪ 1.3

eXercise

Questions in this set

- 9231/11 N25 Q1 ▪ 9 marks
- 9231/12 N25 Q1 ▪ 9 marks
- 9231/14 N25 Q3 ▪ 9 marks
- 9231/11 J25 Q1 ▪ 8 marks
- 9231/13 J25 Q4 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

9231/11 N25 ▪ Q1 ▪ 9 marks

(a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}(8r^3 + 12r^2 + 4r + 3)$ in terms of n, simplifying your answer. **[3]**

(b) Show that $\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$ and hence use the method of differences to find $\sum_{r=1}^{n} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. **[5]**

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. **[1]**

Solution 1

(a)

Mark scheme

- $2n^2(n+1)^2 + 2n(n+1)(2n+1) + 2n(n+1) + 3n = 2n^4 + 8n^3 + 10n^2 + 7n.$

Solution 1

(b)

Mark scheme

- Common denominator: $\dfrac{(r+1)^4 - r^4}{r^4(r+1)^4} = \dfrac{4r^3+6r^2+4r+1}{r^4(r+1)^4}$ (AG). Telescoping: $\sum_{r=1}^{n} = 1 - \dfrac{1}{(n+1)^4}$.

Solution 1

(c)

Mark scheme

- As $n \to \infty$, the sum $\to 1$.

Question 1

9231/12 N25 ▪ Q1 ▪ 9 marks

(a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}(r^3 - r)$ in terms of n, fully factorising your answer. **[3]**

Question 1

9231/12 N25 ▪ Q1 ▪ 9 marks

(b) Express $\dfrac{r+3}{r^3-r}$ in the form $\dfrac{A}{r-1}+\dfrac{B}{r}+\dfrac{C}{r+1}$, where A, B and C are constants to be determined, and hence use the method of differences to find $\displaystyle\sum_{r=2}^{n}\frac{r+3}{r^3-r}$. **[5]**

Question 1

9231/12 N25 ▪ Q1 ▪ 9 marks

(c) Deduce the value of $\displaystyle\sum_{r=2}^{\infty} \frac{r+3}{r^3 - r}$. **[1]**

Solution 1

(a)

Mark scheme

- $\sum r^3 = \left[\frac{n(n+1)}{2}\right]^2$ and $\sum r = \frac{n(n+1)}{2}$, so $\sum_{r=1}^{n}(r^3 - r) = \frac{1}{4}(n-1)n(n+1)(n+2)$.

Solution 1

(b)

Mark scheme

- $\dfrac{r+3}{r(r-1)(r+1)}$: $A=2,\ B=-3,\ C=1$. By differences, $\sum_{r=2}^{n}\dfrac{r+3}{r^3-r}=\frac{3}{2}-\frac{2}{n}+\frac{1}{n+1}$.

Solution 1

(c)

Mark scheme

- As $n \to \infty$, the sum $\to \frac{3}{2}$.

Question 3

9231/14 N25 ▪ Q3 ▪ 9 marks

(a) Show that $\sum_{r=1}^{n}(r+1)(r+2)(r+3) = \frac{1}{4}n(n^3 + bn^2 + cn + d)$, where the constants b, c and d are to be found. **[3]**

(b) Use the method of differences to find $\sum_{r=1}^{n} \frac{2}{(r+1)(r+2)(r+3)}$, expressing your answer in terms of n. **[5]**

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+2)(r+3)}$. **[1]**

Solution 3

(a)

Mark scheme

- $\sum_{r=1}^{n}(r^3+6r^2+11r+6)=\frac{1}{4}n^2(n+1)^2+n(n+1)(2n+1)+\frac{11}{2}n(n+1)+6n=\frac{1}{4}n(n^3+10n^2+35n+50)$. So $b=10$, $c=35$, $d=50$.

Solution 3

(b) Mark scheme

- Partial fractions: $\dfrac{2}{(r+1)(r+2)(r+3)} = \dfrac{1}{r+1} - \dfrac{2}{r+2} + \dfrac{1}{r+3}$. With $f(r) = \frac{1}{r+1}$ the sum telescopes to $\displaystyle\sum_{r=1}^{n} = \frac{1}{6} - \frac{1}{n+2} + \frac{1}{n+3} \left(= \frac{1}{6} - \frac{1}{(n+2)(n+3)} \right)$.

Solution 3

(c)

Mark scheme

- As $n \to \infty$, the sum $\to \frac{1}{6}$.

Question 1

9231/11 J25 ▪ Q1 ▪ 8 marks

(a) Use standard results from the list of formulae (MF19) to show that

$\sum_{r=1}^{n}(2-3r)(5-3r) = an^3 + bn^2 + cn$, where a, b and c are integers to be determined. **[3]**

(b) Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2-3r)(5-3r)}$ in terms of n. **[4]**

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(2-3r)(5-3r)}$. **[1]**

Solution 1

(a)

Mark scheme

- $(2-3r)(5-3r) = 9r^2 - 21r + 10$. So $\sum = 9 \cdot \frac{1}{6}n(n+1)(2n+1) - 21 \cdot \frac{1}{2}n(n+1) + 10n = 3n^3 - 6n^2 + n$. Hence $a = 3,\ b = -6,\ c = 1$.

Solution 1

(b)

Mark scheme

- Partial fractions: $\dfrac{1}{(2-3r)(5-3r)} = \frac{1}{3}\left(\dfrac{1}{2-3r} - \dfrac{1}{5-3r}\right)$. The sum telescopes to $\displaystyle\sum_{r=1}^{n} = -\tfrac{1}{6} + \dfrac{1}{3(2-3n)}$.

Solution 1

(c)

Mark scheme

- As $n \to \infty$, $\dfrac{1}{3(2-3n)} \to 0$, so the sum $\to -\frac{1}{6}$.

Question 4

9231/13 J25 ▪ Q4 ▪ 8 marks

Let $w_r = r(r+1)(r+2)\dots(r+9)$.

(a) Show that $w_{r+1} - w_r = 10(r+1)(r+2)\dots(r+9)$. **[2]**

(b) Given that $u_r = (r+1)(r+2)\dots(r+9)$, find $\sum_{r=1}^{n} u_r$ in terms of n. **[3]**

(c) Given that $v_r = x^{w_{r+1}} - x^{w_r}$, find the set of values of x for which the infinite series $v_1 + v_2 + v_3 + \dots$ is convergent and give the sum to infinity when this exists. **[3]**

Solution 4

(a)

Mark scheme

- $w_{r+1} - w_r = (r+1)(r+2)\ldots(r+10) - r(r+1)\ldots(r+9) = (r+1)(r+2)\ldots(r+9)\big[(r+10) - r\big] = 10(r+1)(r+2)\ldots(r+9)$ (AG).

Solution 4

(b)

Mark scheme

- $\sum_{r=1}^{n} u_r = \frac{1}{10}\sum_{r=1}^{n}(w_{r+1} - w_r) = \frac{1}{10}(w_{n+1} - w_1) = \frac{1}{10}(n+1)(n+2)\ldots(n+10) - 9!$ (since $w_1 = 10!$ and $\frac{10!}{10} = 9!$).

Solution 4

(c)

Mark scheme

- $v_1 + v_2 + \ldots + v_n = x^{w_{n+1}} - x^{w_1}$. For $-1 < x < 1$, $x^{w_{n+1}} \to 0$, so the sum to infinity is $-x^{w_1} = -x^{10!}$. For $x = \pm 1$ every term is 0, so the sum to infinity is 0. Hence the series converges for $-1 \leqslant x \leqslant 1$.