

1. Using $\cosh x = (e^x + e^{-x})/2$, what is $\cosh 0$?
- _____
2. The matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ has characteristic equation $(2-\lambda)^2 - 1 = 0$, giving $\lambda = 1$ or $\lambda = 3$. What is the larger eigenvalue?
- _____
3. The derivative of $\sinh x$ is:
- ☐ $-\cosh x$
- ☐ $\tanh x$
- ☐ $\sinh x$
- ☐ $\cosh x$
4. What is $\int 1/(a^2 + x^2) dx$?
- ☐ $\sin^{-1}(x/a) + C$
- ☐ $a \tan^{-1}(ax) + C$
- ☐ $\ln(a^2 + x^2) + C$
- ☐ $(1/a) \tan^{-1}(x/a) + C$
5. De Moivre's theorem states $(\cos \theta + i \sin \theta)^n$ equals:
- ☐ $\cos \theta^n + i \sin \theta^n$
- ☐ $\cos n\theta + i \sin n\theta$
- ☐ $\cos(\theta/n) + i \sin(\theta/n)$
- ☐ $n(\cos \theta + i \sin \theta)$
6. For $dy/dx + 2y = e^x$, the integrating factor $\mu = e^{\int 2 dx}$ is:
- ☐ $e^{(2x)}$
- ☐ $2x$
- ☐ $e^{(x^2)}$
- ☐ e^x
7. For any x , what does $\cosh^2 x - \sinh^2 x$ equal?
- _____
8. The eigenvalues of a matrix A are found by solving:

☐ $Av = 0$

☐ $\det(A) = 1$

☐ $\det(A - \lambda I) = 0$

☐ $A = 0$

9. The derivative of $\tan^{-1}x$ is $1/(1+x^2)$. What is its value at $x = 0$?

10. In $\int 1/(9 + x^2) \, dx = (1/a)\tan^{-1}(x/a) + C$, what is a ?

A-Level Further Mathematics

1. **1**

$$\cosh 0 = (e^0 + e^0)/2 = (1 + 1)/2 = 1.$$

2. **3**

$$(2-\lambda)^2 = 1 \rightarrow 2-\lambda = \pm 1 \rightarrow \lambda = 1 \text{ or } 3; \text{ the larger is } 3.$$

3. **cosh x**

$$d/dx(\sinh x) = \cosh x \text{ (and } d/dx(\cosh x) = \sinh x).$$

4. **$(1/a) \tan^{-1}(x/a) + C$**

This is the standard inverse-tangent integral.

5. **$\cos n\theta + i \sin n\theta$**

Raising to the power n multiplies the argument by n: $\cos n\theta + i \sin n\theta$.

6. **e^{2x}**

$$\int 2 \, dx = 2x, \text{ so } \mu = e^{2x}.$$

7. **1**

$$\text{The hyperbolic identity is } \cosh^2 x - \sinh^2 x = 1.$$

8. **$\det(A - \lambda I) = 0$**

The characteristic equation $\det(A - \lambda I) = 0$ gives the eigenvalues.

9. **1**

$$1/(1+0^2) = 1.$$

10. **3**

$$a^2 = 9 \text{ so } a = 3; \text{ the integral is } (1/3)\tan^{-1}(x/3) + C.$$