

2 Hyperbolic functions and eigenvalues – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English	Write it 3 times (English)
hyperbolic functions	_____
eigenvector	_____
eigenvalue	_____
characteristic equation	_____

Recall

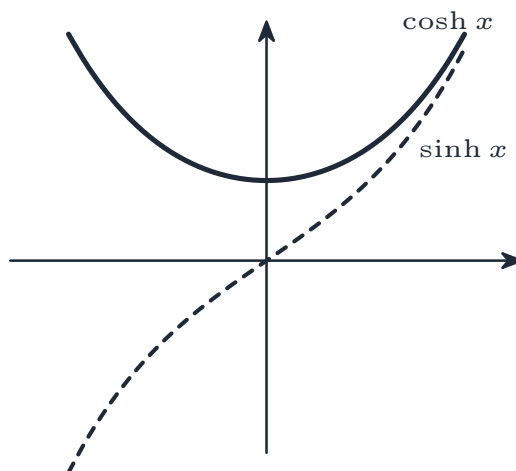
Further Pure 2 adds hyperbolic functions and the eigenvalues of a matrix.

New ideas

Read these first, then do the Practice.

■ 1 Hyperbolic functions

The **hyperbolic functions** are built from e^x :

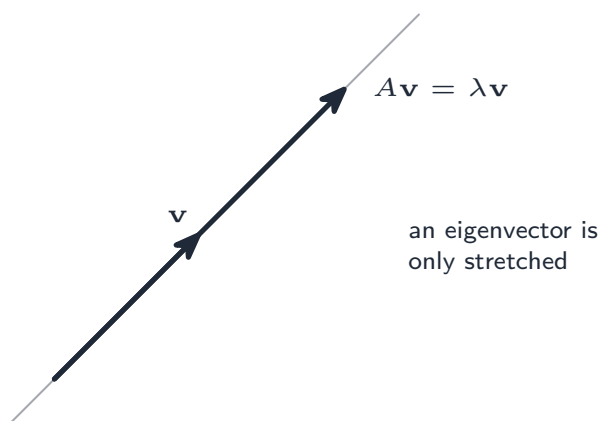


$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

The main identity is $\cosh^2 x - \sinh^2 x = 1$.

■ 2 Eigenvalues

For a square matrix A , an **eigenvector** $\mathbf{v}$ is only stretched (not turned) by A : $A\mathbf{v} = \lambda\mathbf{v}$. The number λ is an **eigenvalue** :



You find λ from the **characteristic equation** $\det(A - \lambda I) = 0$.

Worked example. For $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$: $\det(A - \lambda I) = (3 - \lambda)(2 - \lambda) = 0$, so the eigenvalues are $\lambda = 3$ and $\lambda = 2$.

Watch out: the hyperbolic identity has a **minus** sign $\cosh^2 x - \sinh^2 x = 1$ (unlike $\cos^2 x + \sin^2 x = 1$), and $\cosh x \geq 1$ always. For eigenvalues, subtract λ from the **diagonal** of A before taking the determinant.

Practice

Try these in order.

2.1 Write $\cosh x$ in terms of e^x . [1]

2.2 What is the main hyperbolic identity? [1]

2.3 Find $\cosh 0$. [1]

2.4 What does it mean for $\mathbf{v}$ to be an *eigenvector* of A ? [1]

2.5 Which equation gives the eigenvalues of A ? [1]

2.6 Challenge. Find the eigenvalues of $A = \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}$. [3]