1 Further Pure Mathematics 1 – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English	**Write it 3 times** (English)
Mathematical induction	
Polar coordinates	
Cartesian coordinates	

Recall

In **Part 1** you linked a quadratic's roots to its coefficients, and met the 2×2 matrix. Part 2 adds a way to **prove** a result for every n (induction), and a new way to give a point's position (**polar coordinates**).

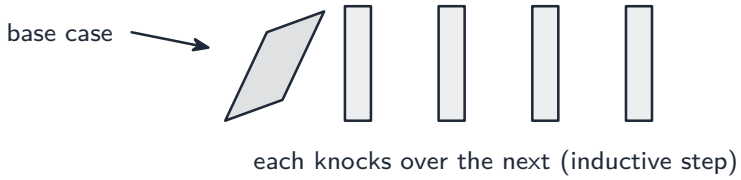
New ideas

■ 1 Proof by induction

Mathematical induction proves a result is true for every positive integer n in two steps:

1. **base case** – show the result is true for $n = 1$;
2. **inductive step** – assume it is true for $n = k$, then prove it for $n = k + 1$.

If both steps work, the result holds for **all** n. It is like a row of dominoes: knock the first over, and each one knocks over the next.

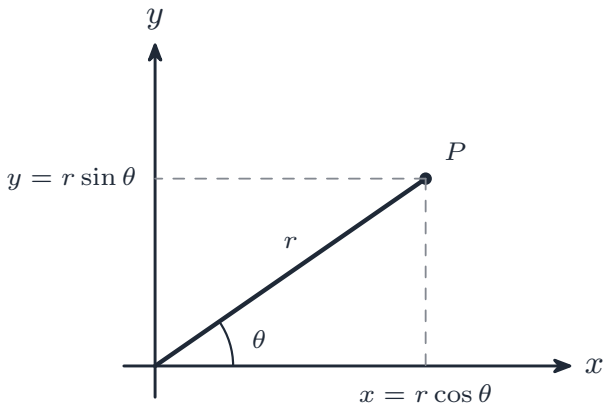


Micro-check. Name the two steps of a proof by induction. [2]

2 Polar coordinates

Polar coordinates give a point by its distance r from the origin and its angle θ. They link to **Cartesian coordinates** by

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad r^2 = x^2 + y^2.$$



Worked example. The polar point $(r, \theta) = (2, 90°)$ has $x = 2\cos 90° = 0$ and $y = 2\sin 90° = 2$.

Micro-check. A point has $x = 3$ and $y = 4$. Find its distance r from the origin. [1]

Watch out: induction needs **both** steps – assuming the result for $n = k$ and then proving it for $n = k + 1$; the base case alone proves nothing. For polar form, $r = \sqrt{x^2 + y^2}$ is always positive, and $x = r\cos\theta$ (not $r\sin\theta$).

Practice

1.1 State, in order, the two steps of a proof by **mathematical induction**. [2]

1.2 A result is being proved by induction. State what you do in the **base case**. [1]

1.3 Convert the polar point $(r, \theta) = (4, 0°)$ to Cartesian coordinates (x, y). [2]

1.4 A point has Cartesian coordinates $(6, 8)$. Find its distance r from the origin. [2]

1.5 Write the formulae linking x and y to r and θ. [2]

1.6 Challenge. Convert the Cartesian point $(3, 4)$ to polar coordinates (r, θ), giving θ to the nearest degree. [3]