

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Vocabulary

Write each word three times.

| English         | Write it 3 times (English) |       |       |
|-----------------|----------------------------|-------|-------|
| roots           | _____                      | _____ | _____ |
| coefficients    | _____                      | _____ | _____ |
| matrix          | _____                      | _____ | _____ |
| determinant     | _____                      | _____ | _____ |
| inverse matrix  | _____                      | _____ | _____ |
| identity matrix | _____                      | _____ | _____ |

Recall

In AS Maths you solved quadratics and worked with their coefficients. Bring this with you:

- a quadratic  $ax^2 + bx + c$  has coefficients  $a$ ,  $b$ ,  $c$ ;
- you solved  $ax^2 + bx + c = 0$  by factorising.

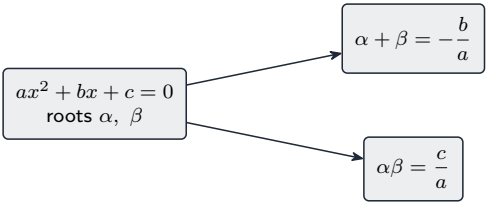
Further Pure 1 links a polynomial’s **roots** to its coefficients, and adds a new object – the **matrix**.

New ideas

■ 1 Roots of a quadratic

For  $ax^2 + bx + c = 0$  with **roots**  $\alpha$  and  $\beta$ , the **coefficients** give the sum and product directly:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$



*Worked example.* For  $x^2 - 5x + 6 = 0$ :  $\alpha + \beta = 5$  and  $\alpha\beta = 6$  (the roots are 2 and 3).

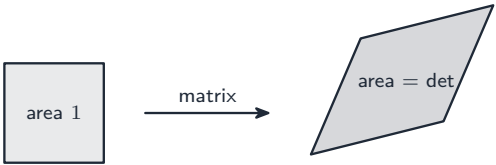
**Micro-check.** For  $x^2 - 7x + 10 = 0$ , write down  $\alpha + \beta$  and  $\alpha\beta$ . [2]

■ 2 Matrices

A **matrix** is a block of numbers. For a  $2 \times 2$  matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , the **determinant** is  $ad - bc$ . When  $ad - bc \neq 0$ , the **inverse matrix** is

$$\frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

The **identity matrix**  $I$  leaves any matrix unchanged. A  $2 \times 2$  matrix maps the unit square to a parallelogram whose area is the determinant.



*Worked example.* The determinant of  $\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$  is  $3 \times 4 - 1 \times 2 = 10$ .

**Micro-check.** Find the determinant of  $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ . [1]

**Watch out:** the sum of the roots is  $-\frac{b}{a}$  – keep the minus sign. For the inverse, swap the diagonal entries  $a$  and  $d$ , negate  $b$  and  $c$ , then divide by the determinant  $ad - bc$  (which must not be 0).

Practice

1.1 For  $x^2 - 8x + 15 = 0$  with roots  $\alpha, \beta$ , write down (a)  $\alpha + \beta$  and (b)  $\alpha\beta$ . [2]

1.2 A quadratic  $2x^2 + 6x + 4 = 0$  has roots  $\alpha, \beta$ . Find  $\alpha + \beta$  and  $\alpha\beta$ . [2]

1.3 Find the determinant of  $\begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$ . [2]

1.4 Find the inverse of  $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ . [2]

1.5 State what the **identity matrix**  $I$  does when it multiplies another matrix. [1]

**1.6 Challenge.** The equation  $x^2 - 6x + 8 = 0$  has roots  $\alpha$  and  $\beta$ . Without solving it, find  $\alpha^2 + \beta^2$ . [2]