4.3 Chi-squared tests

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS goodness of fit • contingency table • independence • chi-squared test • degrees of freedom

Objective

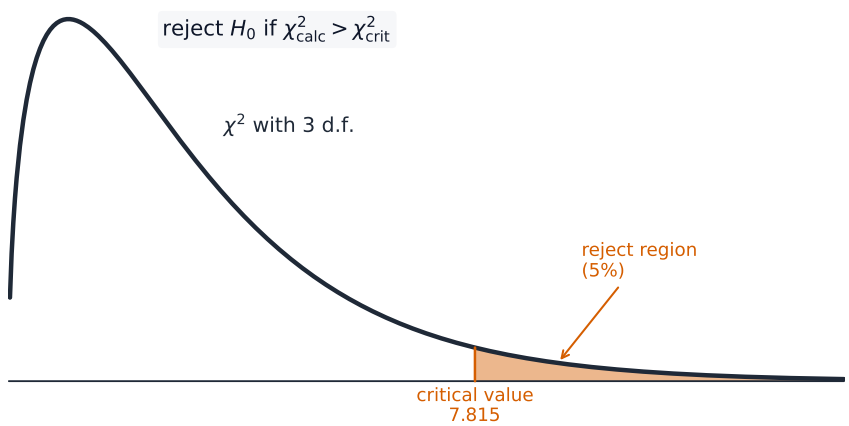
Build the skills to answer exam questions on **chi-squared tests** – **goodness of fit** and **independence** in a **contingency table**.

You must be able to:

- compute expected counts and $\chi^2 = \sum \frac{(O-E)^2}{E}$
- find the degrees of freedom and compare with the critical value
- carry out a goodness-of-fit and an independence test

1 Key knowledge and worked examples

■ Goodness of fit



Reject the model when χ^2 exceeds the critical value (in the shaded tail)

Four equal categories, observed $20, 30, 25, 25$ **(each** $E = 25$**):** $\chi^2 = \frac{25+25+0+0}{25} = 2$. With 3 d.f. the 5% value is 7.815; $2 < 7.815 \to$ do not reject.

■ Key knowledge: Expected contingency counts

For an independence test, $E_{ij} = (\text{row total})(\text{column total})/(\text{grand total})$. Use $(r-1)(c-1)$ degrees of freedom and check expected counts.

■ Worked example: Expected contingency counts

Observed rows are $(18, 12)$ and $(12, 18)$. Each row and column totals 30, so every expected count is $E = 30(30)/60 = 15$. Thus $\chi^2 = 4(3^2/15) = 2.4$, with 1 d.f. The supplied 5% critical value is 3.841; do not reject independence. This does not prove the variables are independent.

2 Practice

2.1 Write down the χ^2 test statistic formula. [1]

2.2 A goodness-of-fit test has 5 categories and no estimated parameters. State the degrees of freedom. [1]

3 Exam-style questions

3.1 In a χ^2 test, a **large** value of χ^2 suggests: [1]

☐ **A** observed and expected agree well
B observed and expected differ a lot
C the sample is small
D the test is invalid

3.2 A die is rolled 120 times, giving the counts:

Face	**1**	**2**	**3**	**4**	**5**	**6**
Observed	15	22	23	19	18	23

Test at the 5% level whether the die is fair ($\chi^2_{\text{crit}} = 11.07$, 5 d.f.). [6]

3.3 For a 3×4 contingency table, state the number of degrees of freedom for a test of independence. [2]