4.1 Continuous random variables

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS probability density function
- cumulative distribution function
- continuous random variables
- median
- percentiles

Objective

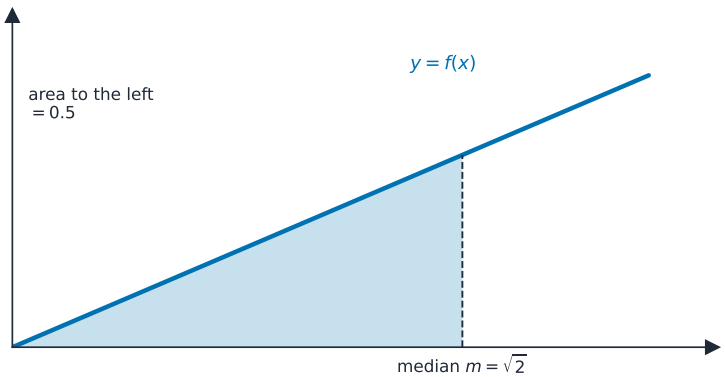
Build the skills to answer exam questions on **continuous random variables** – the **probability density function** , $E(g(X))$, and the **cumulative distribution function** .

You must be able to:

- find a constant / probability from a pdf (possibly piecewise)
- find $E(X)$, $\mathrm{Var}(X)$ and $E(g(X))$ by integration
- find $F(x)$ and use it for the median and percentiles

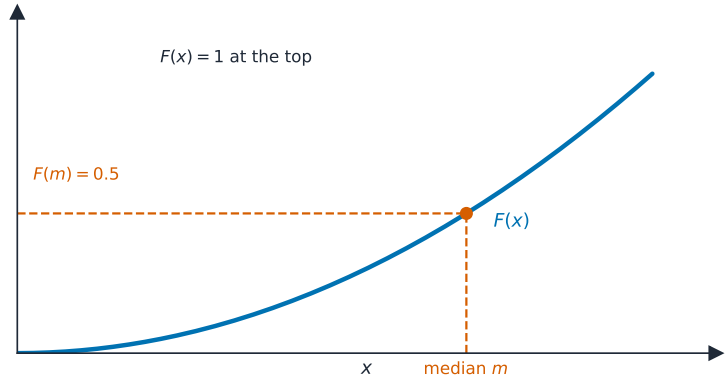
1 Key knowledge and worked examples

■ pdf, cdf and median



median m solves $F(m) = \int_{-\infty}^{m} f(x)\,dx = \frac{1}{2}$

The median is where the area under $f(x)$ to its left is 0.5



$F(x) = P(X \le x) = \int_{-\infty}^{x} f \;\cdot\; 0 \le F \le 1 \;\cdot\; \text{median: } F(m) = \frac{1}{2}$

$F(x)$ rises from 0 to 1; the median is where $F(x) = 0.5$

$f(x) = \frac{1}{2}x$ **on** $[0, 2]$**:** $F(x) = \frac{1}{4}x^2$; median $\frac{1}{4}m^2 = 0.5 \Rightarrow m = \sqrt{2}$.

■ Key knowledge: A transformed expectation

The expectation of a nonlinear function is found by integrating that function against the density. Generally $E[g(X)] \neq g[E(X)]$.

■ Worked example: A transformed expectation

For $f(x) = x/2$ on $[0, 2]$, $E(X) = \int_0^2 x^2/2\,dx = 4/3$. However $E(X^2) = \int_0^2 x^3/2\,dx = 2$, which differs from $(4/3)^2 = 16/9$. Thus $\mathrm{Var}(X) = 2 - 16/9 = 2/9$.

2 Practice

2.1 For $f(x) = \frac{1}{2}x$ on $[0, 2]$, find $E(X)$. [2]

2.2 Write the relationship between $f(x)$ and $F(x)$. [1]

3 Exam-style questions

3.1 The median of a continuous variable solves: [1]

- **A** $f(m) = 0.5$
- **B** $F(m) = 0.5$
- **C** $E(X) = m$
- **D** $F(m) = 1$

3.2 A continuous variable has $f(x) = kx^2$ for $0 \leqslant x \leqslant 3$ (and 0 elsewhere).

(a) Find k. [3]

(b) Find $E(X)$. [3]

3.3 A variable has cdf $F(x) = \dfrac{x^2}{16}$ for $0 \leqslant x \leqslant 4$.

(a) Find the pdf $f(x)$. [2]

(b) Find the median. [2]