3.1 Motion of a projectile

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS motion of a projectile • projectile • trajectory

Objective

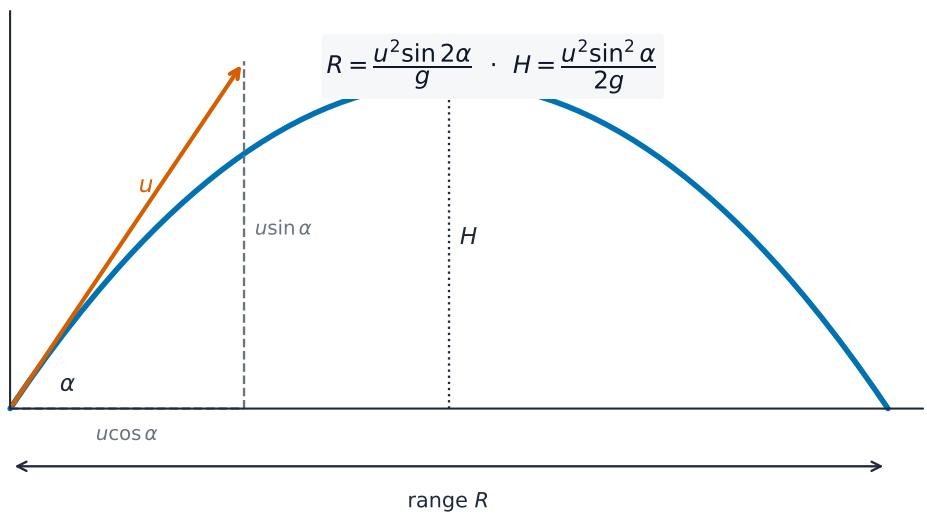
Build the skills to answer exam questions on the **motion of a projectile** – separating horizontal/vertical motion, the **trajectory** equation, range and greatest height. Take $g = 10$ m s^{-2}.

You must be able to:

- use $x = u\cos\alpha\, t$, $y = u\sin\alpha\, t - \frac{1}{2}gt^2$
- eliminate t to get the Cartesian trajectory
- find range $\dfrac{u^2\sin 2\alpha}{g}$ and greatest height $\dfrac{u^2\sin^2\alpha}{2g}$

1 Key knowledge and worked examples

Range and height



The path is a parabola; u splits into a steady horizontal part and a vertical part slowed by gravity

$u = 20$, $\alpha = 30°$: range $= \dfrac{400 \sin 60°}{10} = 34.6$ m; greatest height $= \dfrac{400 \sin^2 30°}{20} = 5$ m.

■ **Key knowledge: Landing above the launch level**

The usual range formula assumes launch and landing at the same height. For a different level, solve the vertical equation for the positive flight time first.

■ **Worked example: Landing above the launch level**

Launch at $u = 20\,\mathrm{m\,s^{-1}}$, angle 30°, with $g = 10$, landing 5 m above launch. The vertical equation is $5 = 10t - 5t^2$, so $(t-1)^2 = 0$ and $t = 1$ s. Then $x = u \cos 30°\, t = 10\sqrt{3}$ m. This height is the trajectory maximum.

2 Practice

2.1 A projectile is launched at 25 m s^{-1} at 40°. Find its initial vertical velocity component. [2]

2.2 Write down the formula for the range on level ground. [1]

3 Exam-style questions

3.1 A ball is projected horizontally at 15 m s^{-1} from a height. After 2 s its horizontal distance is: [1]

- ☐ **A** 15 m
- **B** 30 m
- **C** 20 m
- **D** 7.5 m

3.2 A particle is projected at 30 m s^{-1} at 60° to the horizontal.

(a) Find the time to reach the greatest height. [2]

(b) Find the greatest height reached. [3]

3.3 A stone is thrown at 20 m s^{-1} at 45° from the top of a cliff. It lands in the sea 50 m horizontally away.

(a) Find the time of flight. [3]

(b) Find the height of the cliff above the sea. [3]