2.1 Hyperbolic functions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS hyperbolic functions • logarithmic form

Objective

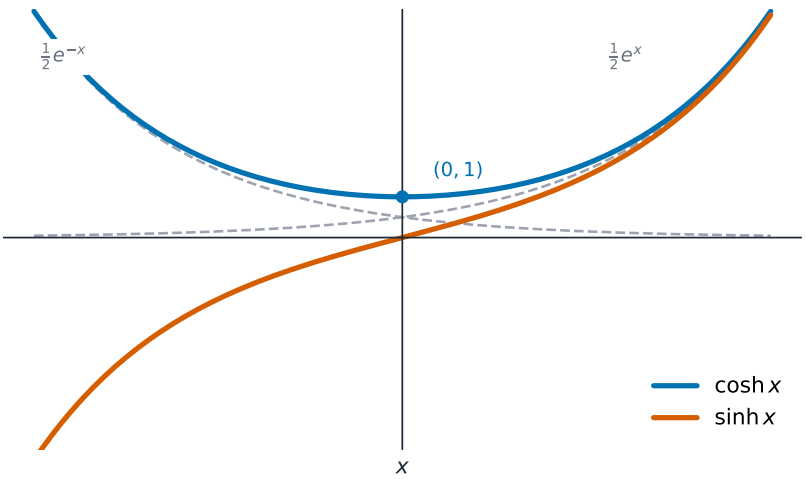
Build the skills to answer exam questions on **hyperbolic functions** – the definitions, identities, and the **logarithmic form** of the inverses.

You must be able to:

- use $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$
- use $\cosh^2 x - \sinh^2 x = 1$ and related identities
- use $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$ and solve equations

1 Key knowledge and worked examples

Identity & solving



$\cosh x = \frac{e^x + e^{-x}}{2}$ · $\sinh x = \frac{e^x - e^{-x}}{2}$ · $\cosh^2 x - \sinh^2 x = 1$

cosh *is the average of* e^x *and* e^{-x}*;* sinh *is half their difference*

$$\cosh^2 x - \sinh^2 x = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{4} = \frac{4}{4} = 1.$$

Solve $\cosh x = 2$**:** $\dfrac{e^x + e^{-x}}{2} = 2 \Rightarrow e^{2x} - 4e^x + 1 = 0 \Rightarrow e^x = 2 \pm \sqrt{3}$, $x = \ln(2 \pm \sqrt{3})$.

■ **Key knowledge: An inverse hyperbolic function**

Derive the logarithmic form from the exponential definition and choose the positive exponential root. This also identifies the valid domain.

■ **Worked example: An inverse hyperbolic function**

For $y = \operatorname{arsinh} x$, $x = (e^y - e^{-y})/2$. Setting $u = e^y > 0$ gives $u^2 - 2xu - 1 = 0$. Thus $u = x + \sqrt{x^2 + 1}$, because the other root is negative. Therefore $y = \ln(x + \sqrt{x^2 + 1})$, valid for every real x.

2 Practice

2.1 Write down the value of $\cosh 0$. [1]

2.2 Evaluate $\sinh^{-1} 0$ using the logarithmic form. [1]

3 Exam-style questions

3.1 The identity satisfied by hyperbolic functions is: [1]

☐ **A** $\cosh^2 x + \sinh^2 x = 1$
B $\cosh^2 x - \sinh^2 x = 1$
C $\sinh^2 x - \cosh^2 x = 1$
D $\cosh^2 x - \sinh^2 x = -1$

3.2 Solve the equation $\cosh x = 3$, giving exact answers. [4]

3.3 (a) Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$. [4]

(b) Hence evaluate $\sinh^{-1}(\frac{3}{4})$ as a single logarithm. [2]