

1.7 Proof by induction

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS proof by induction • inductive proof

Objective

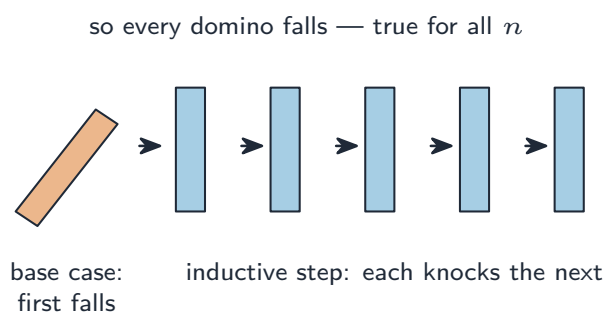
Build the skills to answer exam questions on **proof by induction** – the base case, the inductive step, and a clear concluding statement, for series, divisibility and matrices.

You must be able to:

- prove a summation formula by induction
- prove a divisibility result by induction
- write the base case, inductive step and conclusion clearly

1 Key knowledge and worked examples

■ The two steps



Base case topples the first; the inductive step makes each one knock over the next

Prove $\sum_{r=1}^n r = \frac{1}{2}n(n+1)$. Base $n = 1$: $1 = \frac{1}{2}(1)(2)$ ✓. Assume true for k ; then $\sum_1^{k+1} = \frac{1}{2}k(k+1) + (k+1) = \frac{1}{2}(k+1)(k+2)$ – the formula at $n = k+1$. So true for all n .

■ **Key knowledge: The inductive implication**

A proof needs a verified base case and an implication from the claim for k to the claim for $k+1$. Assuming the final result for both is circular.

■ **Worked example: The inductive implication**

Prove $1 + 3 + \cdots + (2n - 1) = n^2$. The case $n = 1$ gives $1 = 1^2$. Assume the sum to k is k^2 . Adding the next term gives $S_{k+1} = k^2 + (2k + 1) = (k + 1)^2$. Hence the claim holds for every positive integer n by induction.

2 Practice

2.1 State the two steps of a proof by induction. [2]

2.2 Verify the base case $n = 1$ for $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$. [1]

3 Exam-style questions

3.1 In an inductive proof, the inductive step assumes the result for $n = k$ and proves it for: [1]

☐

A $n = 1$

B $n = k - 1$

C $n = k + 1$

D all n at once

3.2 Prove by induction that $\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2)$. [5]

3.3 Prove by induction that $3^{2n} - 1$ is divisible by 8 for all positive integers n . [5]
