1.6 Vectors

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

KEY WORDS plane • vectors • vector product

Objective

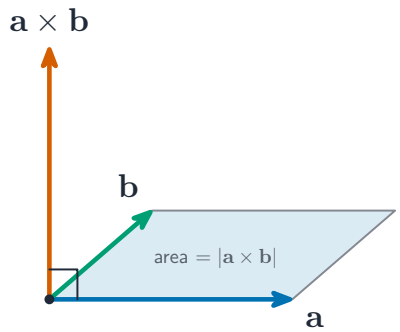
Build the skills to answer exam questions on **vectors** in 3-D – the **vector product**, equations of a **plane**, and distances/angles between lines and planes.

You must be able to:

- compute $\mathbf{a} \times \mathbf{b}$ and use it for a normal/area
- write a plane as $\mathbf{r} \cdot \mathbf{n} = p$ or $ax + by + cz = d$
- find the angle between, and distance from, lines and planes

1 Key knowledge and worked examples

Vector product & plane



$\mathbf{a} \times \mathbf{b}$ *is perpendicular to both; its length is the area of the parallelogram*

$\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} + \mathbf{k}$: $\mathbf{a} \times \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

Plane through A with normal $\mathbf{n}$: $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$.

Key knowledge: A point-to-plane distance

For a plane $\mathbf{n} \cdot \mathbf{r} = d$, perpendicular distance from $\mathbf{p}$ is $|\mathbf{n} \cdot \mathbf{p} - d|/|\mathbf{n}|$. The absolute value makes distance non-negative.

Worked example: A point-to-plane distance

Find the distance from $(1, 2, 3)$ to $2x - y + 2z = 4$. The normal is $(2, -1, 2)$ with magnitude 3. Therefore $D = |2(1) - 2 + 2(3) - 4|/3 = 2/3$. A point on the plane would make the numerator zero.

2 Practice

2.1 Find $\mathbf{a} \times \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{k}$. [3]

2.2 Write the plane through $(1, 2, 3)$ with normal $\mathbf{n} = (1, 1, 1)$ in the form $ax + by + cz = d$. [2]

3 Exam-style questions

3.1 $\mathbf{a} \times \mathbf{b}$ is a vector that is: [1]

- ☐ **A** parallel to $\mathbf{a}$
- **B** perpendicular to both $\mathbf{a}$ and $\mathbf{b}$
- **C** equal to $\mathbf{a} \cdot \mathbf{b}$
- **D** of length $|\mathbf{a}||\mathbf{b}| \cos\theta$

3.2 Points $P(1,0,2)$, $Q(3,1,1)$ and $R(2,-2,4)$ are given.

(a) Find $\overrightarrow{PQ} \times \overrightarrow{PR}$, and hence the area of triangle PQR. [4]

(b) Find the equation of the plane PQR, and the perpendicular distance from the origin to it. [4]

3.3 Find the acute angle between the planes $x + 2y + 2z = 5$ and $2x - y + 2z = 1$. [4]