1.1 Roots of polynomial equations

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS roots • coefficients • symmetric functions • roots of polynomial equations • substitution

Objective

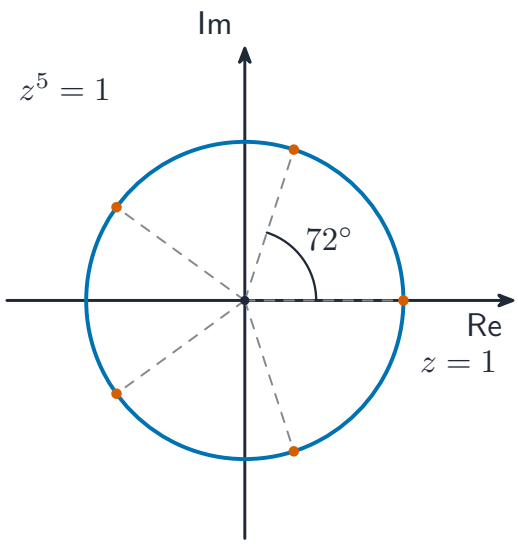
Build the skills to answer exam questions on **roots of polynomial equations** – the relations between roots and **coefficients**, **symmetric functions**, and forming a new equation by **substitution**.

You must be able to:

- use $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$ for quadratics and cubics
- evaluate symmetric functions such as $\sum \alpha^2$
- form an equation whose roots are a transformation of the originals

1 Key knowledge and worked examples

Roots and coefficients



The roots of a polynomial are tied to its coefficients by $\sum \alpha$, $\sum \alpha\beta$, ...

$x^2 - 5x + 6 = 0$ **has roots** α, β**. Find the equation with roots** $\alpha + 1, \beta + 1$**.** $\alpha + \beta = 5$, $\alpha\beta = 6$; new sum $= 7$, new product $= \alpha\beta + \alpha + \beta + 1 = 12$. So $x^2 - 7x + 12 = 0$.

$\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta$**.**

■ **Key knowledge: A polynomial from transformed roots**

If the old roots satisfy a polynomial, substitute the inverse root transformation into that polynomial to obtain an equation for the new roots.

■ **Worked example: A polynomial from transformed roots**

If α, β solve $x^2 - 3x + 1 = 0$, find the equation with roots $\alpha + 2, \beta + 2$. Set $y = x + 2$, so $x = y - 2$. Substitution gives $(y - 2)^2 - 3(y - 2) + 1 = 0$, hence $y^2 - 7y + 11 = 0$. The new root sum is $3 + 4 = 7$, providing a check.

2 Practice

2.1 For $x^2 - 7x + 10 = 0$ with roots α, β, write down $\alpha + \beta$ and $\alpha\beta$. [2]

2.2 Hence find $\alpha^2 + \beta^2$. [2]

3 Exam-style questions

3.1 For $2x^3 - 3x^2 + 4x - 5 = 0$ with roots α, β, γ, the value of $\alpha + \beta + \gamma$ is: [1]

- ☐ **A** $\frac{3}{2}$
- **B** $-\frac{3}{2}$
- **C** 2
- **D** $\frac{5}{2}$

3.2 The cubic $x^3 - 4x^2 + 5x - 2 = 0$ has roots α, β, γ.

(a) Write down $\sum \alpha$, $\sum \alpha\beta$ and $\alpha\beta\gamma$. [3]

(b) Find $\alpha^2 + \beta^2 + \gamma^2$. [3]

3.3 The equation $x^2 + 3x - 1 = 0$ has roots α, β. Find a quadratic equation with integer coefficients whose roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$. [4]