4.2 Inference using normal and t-distributions

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS confidence interval • degrees of freedom

Objective

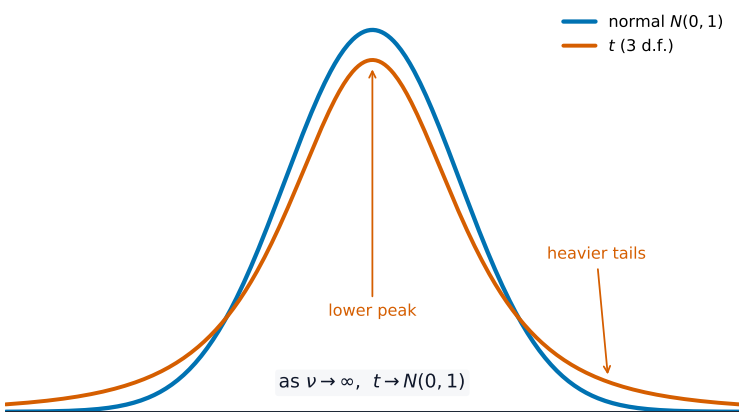
Build the skills to answer exam questions on **inference using the normal and t-distributions** t – small-sample **confidence intervals** and **t-tests** t .

You must be able to:

- find a confidence interval $\bar{x} \pm t\frac{s}{\sqrt{n}}$ (with $n-1$ d.f.)
- carry out a one-sample t-test
- carry out a two-sample / paired t-test (pooled variance where needed)

1 Key knowledge and worked examples

■ Small-sample interval



For a small sample the t-distribution is flatter with heavier tails, so its critical values are larger

$n = 10$, $\bar{x} = 50$, $s = 4$, 95% ($t = 2.262$, 9 **d.f.**): $50 \pm 2.262\frac{4}{\sqrt{10}} = 50 \pm 2.86 =$ (47.1, 52.9).

■ Key knowledge: A paired t-test

For paired measurements, work with the differences, not two independent samples. Use $n-1$ degrees of freedom and state the normal-differences assumption for a small sample.

■ Worked example: A paired t-test

Ten independent pairs have mean difference $\bar{d} = 2.4$ and difference standard deviation $s_d = 3.0$. Testing $H_0 : \mu_d = 0$ against $H_1 : \mu_d > 0$ gives $t = \bar{d}/(s_d/\sqrt{n}) = 2.530$. The supplied 5% one-tailed critical value for 9 d.f. is 1.833, so reject H_0. The evidence concerns the mean paired difference.

2 Practice

2.1 State the number of degrees of freedom for a one-sample t-test with $n = 12$. [1]

2.2 State when a t-distribution is used instead of the normal. [1]

3 Exam-style questions

3.1 A 95% confidence interval based on a small sample uses a t value that is, compared with 1.96: [1]

☐ **A** smaller
B larger
C equal
D zero

3.2 A sample of 8 measurements has $\bar{x} = 20.5$ and $s = 1.2$.

(a) Find a 95% confidence interval for the population mean (use $t = 2.365$, 7 d.f.). [3]

(b) State one assumption needed for this interval to be valid. [1]

3.3 A machine should produce rods of mean length 25 cm. A sample of 6 rods has $\bar{x} = 25.4$ cm and $s = 0.5$ cm. Test at the 5% level (two-tailed, $t_{\text{crit}} = 2.571$) whether the mean differs from 25. [5]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $n - 1 = 11$

2.2

- when the sample is small and the population variance is unknown

3.1 B

- The small-sample t value is larger than 1.96

3.2

(a)

- $20.5 \pm 2.365\dfrac{1.2}{\sqrt{8}} = 20.5 \pm 2.365(0.4243) = 20.5 \pm 1.00$; (19.5, 21.5)

(b)

- the population is (approximately) normally distributed

3.3

- $H_0 : \mu = 25$, $H_1 : \mu \neq 25$; $t = \dfrac{25.4 - 25}{0.5/\sqrt{6}} = \dfrac{0.4}{0.2041} = 1.96$; compare with 2.571; since $1.96 < 2.571$, do not reject H_0: no evidence the mean differs from 25