

3.3 Circular motion

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS angular speed • centripetal acceleration • vertical circle
 • horizontal circle • circular motion

Objective

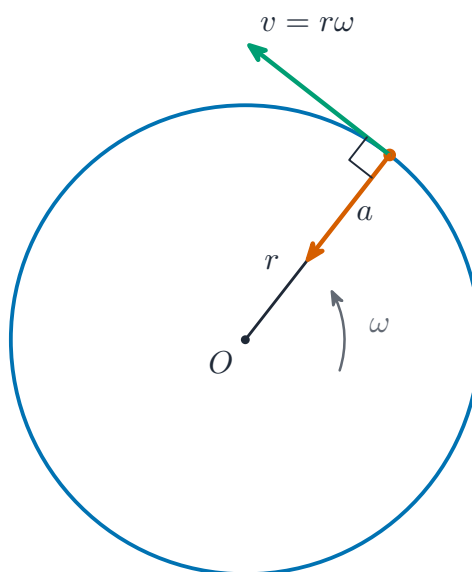
Build the skills to answer exam questions on **circular motion** – **angular speed**, **centripetal acceleration**, the **conical pendulum**, and motion in a **vertical circle**. Take $g = 10 \text{ m s}^{-2}$.

You must be able to:

- use $v = r\omega$ and $a = r\omega^2 = \frac{v^2}{r}$
- analyse a horizontal circle (conical pendulum)
- use energy + the centripetal equation for a vertical circle

1 Key knowledge and worked examples

■ Centripetal force



Velocity points along the tangent; acceleration points inward to the centre

$r = 2 \text{ m}$, $\omega = 3 \text{ rad s}^{-1}$: $v = r\omega = 6 \text{ m s}^{-1}$; $a = r\omega^2 = 18 \text{ m s}^{-2}$.

Centripetal force $= ma = \frac{mv^2}{r}$, provided by tension, friction or a component of weight.

■ **Key knowledge: A vertical-circle contact condition**

Write the radial force equation at the top before using energy. A light string can pull, not push; its tension must stay non-negative.

■ **Worked example: A vertical-circle contact condition**

For a particle on a string of length 1 m, at the top $T + mg = mv^2/r$. At limiting tension $T = 0$, $v_{\text{top}}^2 = gr = 10$. From bottom to top, energy gives $v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gr = 50$. Thus the limiting bottom speed is $\sqrt{50} \approx 7.07 \text{ m s}^{-1}$.

2 Practice

2.1 A particle moves in a circle of radius 0.5 m at 4 m s^{-1} . Find its acceleration. [2]

2.2 Convert an angular speed of 120 rev/min to rad s^{-1} . [2]

3 Exam-style questions

3.1 The centripetal acceleration of a particle moving at speed v in a circle of radius r is: [1]

☐

- A $\frac{v}{r}$
B $\frac{v^2}{r}$
C $\frac{vr}{r}$
D $\frac{r}{v^2}$

3.2 A particle of mass 0.2 kg is attached to a string of length 0.5 m and whirled in a horizontal circle on a smooth table at 3 rev s^{-1} .

(a) Find the angular speed in rad s^{-1} . [2]

(b) Find the tension in the string. [3]

3.3 A conical pendulum has a bob of mass 0.4 kg on a string of length 1 m, which makes an angle of 30° with the vertical.

(a) Find the tension in the string. [3]

(b) Find the angular speed of the bob. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $a = \frac{v^2}{r} = \frac{16}{0.5} = 32 \text{ m s}^{-2}$

2.2

- $120 \text{ rev/min} = 2 \text{ rev/s} = 2 \times 2\pi = 4\pi = 12.6 \text{ rad s}^{-1}$

3.1 B

- $a = \frac{v^2}{r}$ (also $= r\omega^2$)

3.2

(a)

- $\omega = 3 \times 2\pi = 6\pi = 18.8 \text{ rad s}^{-1}$

(b)

- $T = mr\omega^2 = 0.2 \times 0.5 \times (6\pi)^2 = 0.2 \times 0.5 \times 355.3 = 35.5 \text{ N}$

3.3

(a)

- vertical: $T \cos 30^\circ = mg = 4 \Rightarrow T = \frac{4}{0.866} = 4.62 \text{ N}$

(b)

- horizontal: $T \sin 30^\circ = mr\omega^2$ with $r = 1 \times \sin 30^\circ = 0.5$; $4.62 \times 0.5 = 0.4 \times 0.5 \times \omega^2 \Rightarrow \omega^2 = 11.55 \Rightarrow \omega = 3.40 \text{ rad s}^{-1}$