

3.1 Motion of a projectile

Name: _____ Class: _____ Date: _____

Total: 15 marks

KEY WORDS motion of a projectile • projectile • trajectory

Objective

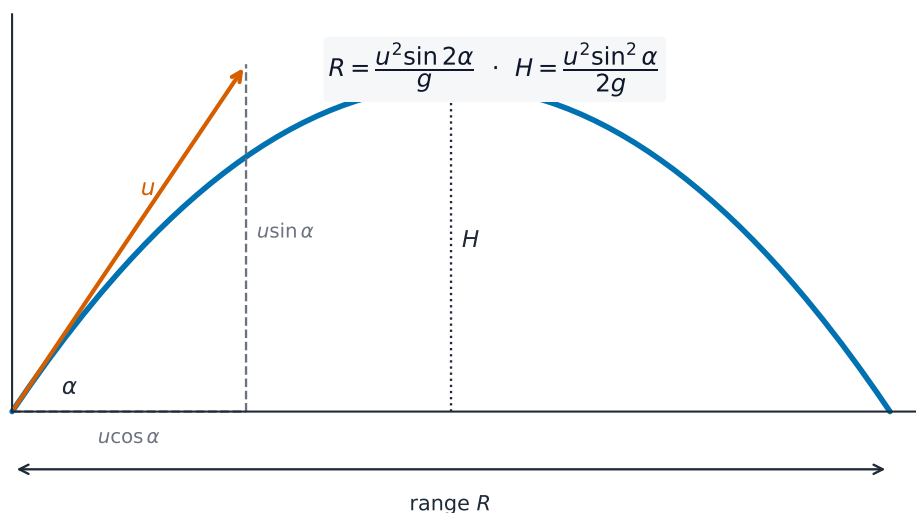
Build the skills to answer exam questions on the **motion of a projectile** – separating horizontal/vertical motion, the **trajectory** equation, range and greatest height. Take $g = 10 \text{ m s}^{-2}$.

You must be able to:

- use $x = u \cos \alpha t$, $y = u \sin \alpha t - \frac{1}{2}gt^2$
- eliminate t to get the Cartesian trajectory
- find range $\frac{u^2 \sin 2\alpha}{g}$ and greatest height $\frac{u^2 \sin^2 \alpha}{2g}$

1 Key knowledge and worked examples

■ Range and height



The path is a parabola; u splits into a steady horizontal part and a vertical part slowed by gravity

$u = 20$, $\alpha = 30^\circ$: range = $\frac{400 \sin 60^\circ}{10} = 34.6$ m; greatest height = $\frac{400 \sin^2 30^\circ}{20} = 5$ m.

■ **Key knowledge: Landing above the launch level**

The usual range formula assumes launch and landing at the same height. For a different level, solve the vertical equation for the positive flight time first.

■ **Worked example: Landing above the launch level**

Launch at $u = 20 \text{ m s}^{-1}$, angle 30° , with $g = 10$, landing 5 m above launch. The vertical equation is $5 = 10t - 5t^2$, so $(t - 1)^2 = 0$ and $t = 1$ s. Then $x = u \cos 30^\circ t = 10\sqrt{3}$ m. This height is the trajectory maximum.

2 Practice

2.1 A projectile is launched at 25 m s^{-1} at 40° . Find its initial vertical velocity component. [2]

2.2 Write down the formula for the range on level ground. [1]

3 Exam-style questions

3.1 A ball is projected horizontally at 15 m s^{-1} from a height. After 2 s its horizontal distance is: [1]

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- A 15 m
- B 30 m
- C 20 m
- D 7.5 m

3.2 A particle is projected at 30 m s^{-1} at 60° to the horizontal.

(a) Find the time to reach the greatest height. [2]

(b) Find the greatest height reached. [3]

3.3 A stone is thrown at 20 m s^{-1} at 45° from the top of a cliff. It lands in the sea 50 m horizontally away.

(a) Find the time of flight. [3]

(b) Find the height of the cliff above the sea. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $25 \sin 40^\circ = 25 \times 0.643 = 16.1 \text{ m s}^{-1}$

2.2

- $\frac{u^2 \sin 2\alpha}{g}$

3.1 B

- $x = 15 \times 2 = 30 \text{ m}$

3.2

(a)

- at greatest height the vertical velocity is 0: $0 = 30 \sin 60^\circ - 10t \Rightarrow t = \frac{25.98}{10} = 2.60 \text{ s}$

(b)

- height = $\frac{u^2 \sin^2 \alpha}{2g} = \frac{900 \times 0.75}{20} = 33.75 \text{ m}$

3.3

(a)

- horizontal: $50 = 20 \cos 45^\circ t = 14.14t \Rightarrow t = 3.54 \text{ s}$

(b)

- take up positive with launch at $y = 0$; at landing $y = -h$: $-h = 20 \sin 45^\circ(3.54) - 5(3.54)^2$; $= 50.0 - 62.5 = -12.5$, so the cliff is $h = 12.5 \text{ m}$ above the sea