

1.5 Polar coordinates

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS polar coordinates • polar curves
• cartesian coordinates

Objective

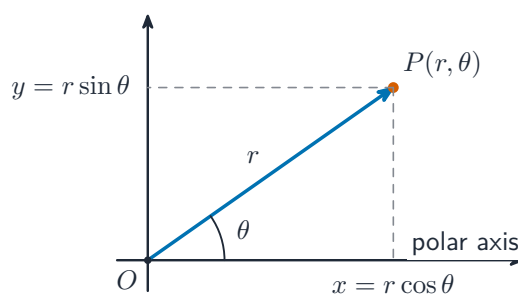
Build the skills to answer exam questions on **polar coordinates** – converting to/from Cartesian, sketching **polar curves**, and the **sector area** $\frac{1}{2} \int r^2 d\theta$.

You must be able to:

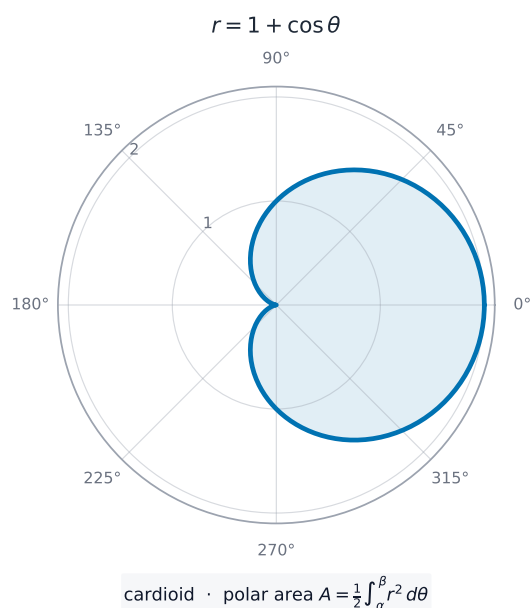
- convert between (r, θ) and (x, y)
- sketch a simple polar curve (e.g. a cardioid)
- find an area with $\frac{1}{2} \int r^2 d\theta$

1 Key knowledge and worked examples

■ Conversion & area



$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2$$



Polar curves are traced as the angle varies

Convert $r = 4 \cos \theta$: $\times r \rightarrow r^2 = 4r \cos \theta$, so $x^2 + y^2 = 4x$, i.e. $(x - 2)^2 + y^2 = 4$ (circle).

Area: $A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$.

■ Key knowledge: Polar area limits

Polar area is $A = \frac{1}{2} \int r^2 d\theta$. Choose limits that trace the required region once; squaring the radius does not fix incorrect limits.

■ Worked example: Polar area limits

For $r = 2 \cos \theta$, the circle is traced once for $-\pi/2 \leq \theta \leq \pi/2$. Thus $A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} 4 \cos^2 \theta d\theta = \pi$. In Cartesian form it is $(x - 1)^2 + y^2 = 1$, whose area confirms the integral.

2 Practice

2.1 Convert the point $(r, \theta) = (2, \frac{\pi}{3})$ to Cartesian coordinates. [2]

2.2 Write down the formula for the area of a polar sector. [1]

3 Exam-style questions

3.1 The polar equation $r = 2 \sin \theta$ represents: [1]

☐

A a line

B a circle

C a parabola

D a spiral

3.2 A curve has polar equation $r = 3 + 2 \cos \theta$ for $0 \leq \theta < 2\pi$ (a convex limaçon).

(a) Write down the greatest and least values of r , and state the value of θ at which each occurs. Explain why the curve never passes through the pole. [3]

(b) Show that the area enclosed by the curve is 11π . [5]

3.3 Convert $r = \frac{2}{1 + \cos \theta}$ to Cartesian form. [4]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $x = 2 \cos \frac{\pi}{3} = 1, y = 2 \sin \frac{\pi}{3} = \sqrt{3}$

2.2

- $A = \frac{1}{2} \int r^2 d\theta$

3.1 B

- $r = 2 \sin \theta \Rightarrow r^2 = 2r \sin \theta \Rightarrow x^2 + y^2 = 2y$, a circle

3.2

(a)

- $\cos \theta$ runs from -1 to 1 , so $r_{\max} = 5$ at $\theta = 0$ and $r_{\min} = 1$ at $\theta = \pi$; the least value is $1 > 0$, so r is never 0 and the curve misses the pole

(b)

- $(3 + 2 \cos \theta)^2 = 9 + 12 \cos \theta + 4 \cos^2 \theta$, and $4 \cos^2 \theta = 2 + 2 \cos 2\theta$, so the integrand is $11 + 12 \cos \theta + 2 \cos 2\theta$; $A = \frac{1}{2} [11\theta + 12 \sin \theta + \sin 2\theta]_0^{2\pi}$; both sine terms vanish at 0 and 2π ; $A = \frac{1}{2}(22\pi) = 11\pi$

3.3

- $r + r \cos \theta = 2 \Rightarrow r = 2 - x$ (since $r \cos \theta = x$); $r^2 = (2 - x)^2 \Rightarrow x^2 + y^2 = 4 - 4x + x^2$; $y^2 = 4 - 4x$, i.e. $y^2 = -4(x - 1)$