

1.4 Matrices

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS transformations • rotation • inverse • enlargement
• determinant

Objective

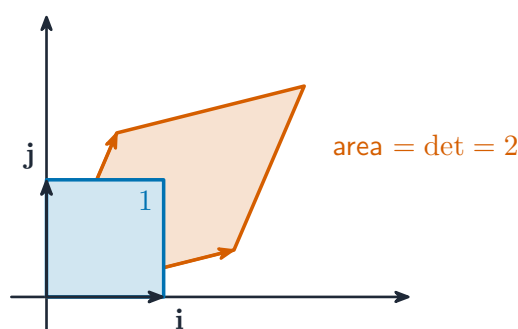
Build the skills to answer exam questions on **matrices** – multiplication, the **determinant** and **inverse**, and 2×2 matrices as **transformations**.

You must be able to:

- multiply matrices and use I
- find $\det A$ and A^{-1} for a 2×2 matrix; use $(AB)^{-1} = B^{-1}A^{-1}$
- identify the transformation a matrix represents (and its area scale factor)

1 Key knowledge and worked examples

■ Inverse & transformation



A matrix maps the unit square to a parallelogram whose area equals $\det A$

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}; \det A = 12 - 2 = 10; A^{-1} = \frac{1}{10} \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix}.$$

Transformations: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is a 90° rotation; $\det = 1$ (area preserved).

■ **Key knowledge: Order in matrix composition**

For column vectors, $AB\mathbf{v}$ applies B first and then A . Matrix multiplication is generally not commutative.

■ **Worked example: Order in matrix composition**

Let $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then $AB = \begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}$, whereas $BA = \begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}$. On $(1, 0)$ these give $(0, 1)$ and $(0, 2)$ respectively, showing that the order changes the result.

2 Practice

2.1 Find $\det \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix}$. [1]

2.2 Find the inverse of $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. [2]

3 Exam-style questions

3.1 The matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ represents: [1]

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A a rotation

B an enlargement, scale factor 2

C a reflection

D a shear

3.2 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$.

(a) Find AB . [2]

(b) Find $\det(AB)$ and verify it equals $\det A \times \det B$. [3]

3.3 The transformation T has matrix $M = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$.

(a) Find the area scale factor of T . [1]

(b) Find M^{-1} and hence the image-to-object matrix that undoes T . [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\det = 5(4) - 2(3) = 14$

2.2

- $\det = 2 - 1 = 1$; inverse = $\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$

3.1 B

- A scalar multiple of I is an enlargement; here scale factor 2

3.2

(a)

- $AB = \begin{pmatrix} 1 \cdot 2 + 2 \cdot 1 & 1 \cdot 0 + 2 \cdot 3 \\ 3 \cdot 2 + 4 \cdot 1 & 3 \cdot 0 + 4 \cdot 3 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 10 & 12 \end{pmatrix}$

(b)

- $\det(AB) = 48 - 60 = -12$; $\det A = -2$, $\det B = 6$, product = -12 ✓

3.3

(a)

- area scale factor = $\det M = 6$

(b)

- $M^{-1} = \frac{1}{6} \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$; this matrix undoes T