

Exercise walk-through

4.3

Chi-squared tests

A-Level Further Mathematics

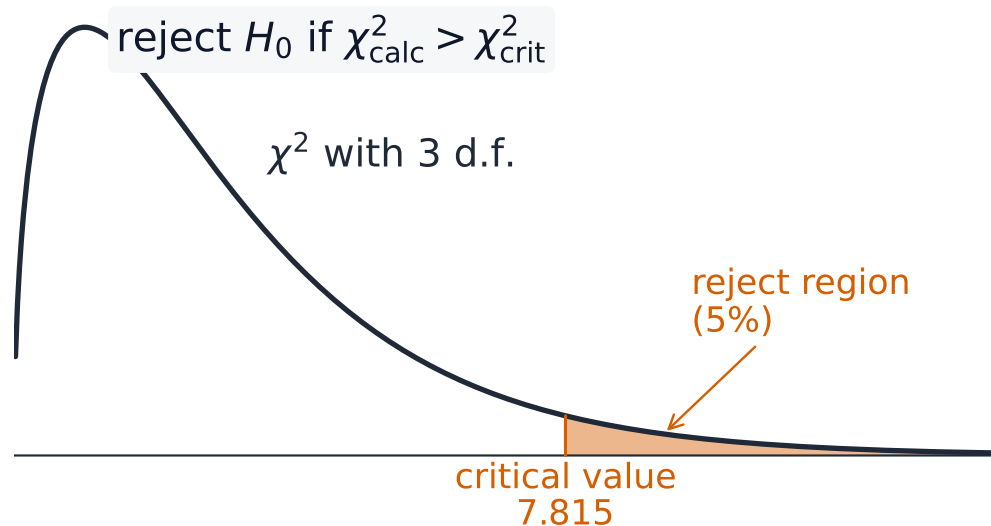
Chi-squared tests

Compare observed counts O with expected E :

$$\chi^2 = \sum \frac{(O - E)^2}{E}.$$

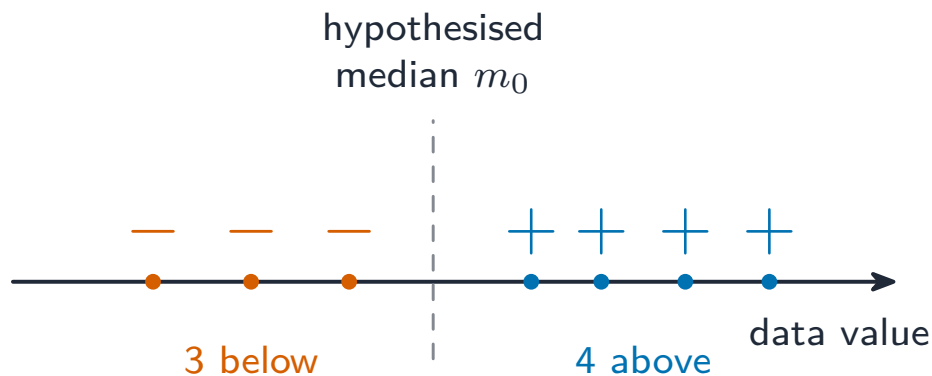
Large $\chi^2 \Rightarrow$ reject H_0 . Degrees of freedom = categories -1 , minus any estimated parameters.

Chi-squared tests, in detail



The test rejects the model when χ^2 exceeds the critical value, landing in the shaded 5%

Non-parametric tests



No assumption that the data is normal:

- **sign test** – counts above/below a median
- **Wilcoxon** signed-rank & rank-sum – use the sizes too

Worked example – a sign test

Ten values are tested against H_0 : median = 5. Nine fall above it. Is that evidence against H_0 ?

- under H_0 , each value is above with probability 0.5, so the count $X \sim B(10, 0.5)$
- $P(X \geq 9) = \binom{10}{9}(0.5)^{10} + \binom{10}{10}(0.5)^{10}$
- $= \frac{10 + 1}{1024} = 0.0107$
- $0.0107 < 0.05$, so **reject** H_0 at the 5% level

The whole test is a binomial tail. Note it uses *only* the signs – discard how far above the median each value was, and a great deal of information with it.

The three non-parametric tests, and what each uses

sign test

uses only **which side** of the hypothesised median each value falls – the weakest, and the most assumption-free

Wilcoxon signed-rank

the **matched-pairs** test: uses the **sizes** of the differences as well as their signs, so it is more powerful

Wilcoxon rank-sum

for **two independent** samples – ranks all the data together and compares the rank totals

when to use them

when normality cannot be assumed, or the data are ordinal

goodness of fit

χ^2 compares observed with expected frequencies; a **contingency table** tests independence

Matched pairs → signed-rank. Two independent samples → rank-sum. Choosing the wrong one loses the whole question, so read for **paired or not** first.

Exercises 2.1·2.2

2.1 Write down the χ^2 test statistic formula.

[1]

2.2 A goodness-of-fit test has 5 categories and no estimated parameters. State the degrees of freedom.

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2.1

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2.2

- $5 - 1 = 4$

Exercise 3.1

3.1 In a χ^2 test, a **large** value of χ^2 suggests:

[1]

☐

A observed and expected agree well

B observed and expected differ a lot

C the sample is small

D the test is invalid

Exercise 3.1

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C the sample is small

D the test is invalid

3.1 B

- A large χ^2 means observed counts differ greatly from expected

Exercises 3.2-3.3

3.2 A die is rolled 120 times, giving the counts:

Face	1	2	3	4	5	6
Observed	15	22	23	19	18	23

Test at the 5% level whether the die is fair ($\chi^2_{\text{crit}} = 11.07$, 5 d.f.).

[6]

3.3 For a 3×4 contingency table, state the number of degrees of freedom for a test of independence.

[2]

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[6]

3.2

- each $E = \frac{120}{6} = 20$; $\chi^2 = \frac{(15 - 20)^2 + (22 - 20)^2 + (23 - 20)^2 + (19 - 20)^2 + (18 - 20)^2 + (23 - 20)^2}{20}$
 $= \frac{25 + 4 + 9 + 1 + 4 + 9}{20} = \frac{52}{20} = 2.6$; $2.6 < 11.07$; do not reject – no evidence the die is unfair

3.3 For a 3×4 contingency table, state the number of degrees of freedom for a test of independence.

[2]

3.3

- degrees of freedom $= (3 - 1)(4 - 1) = 6$