

Exercise walk-through

3.6

Momentum

A-Level Further Mathematics

A bounce needs a system and a direction

Choose the system and contact direction

A **collision** brings two low-friction carts into contact. Their velocities change, but their combined **momentum** stays constant if the external impulse during contact is negligible.

A ball hitting a fixed wall is different: the wall gives the ball an impulse. Define your system before writing a conservation equation.

Decide which momentum is conserved

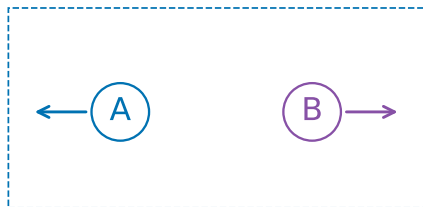
Choose the system and contact direction

Linear momentum is a vector: $\mathbf{p} = m\mathbf{v}$. The law is **conservation of linear momentum**, often shortened to **conservation of momentum**. It applies to a system with zero resultant external impulse, or to a component with zero external impulse in that direction.

For two smooth spheres on a smooth horizontal plane, their mutual impulses cancel. For the sphere alone at a fixed wall, normal momentum changes; the wall belongs outside that one-sphere system.

Conservation requires a stated system and a justified momentum component

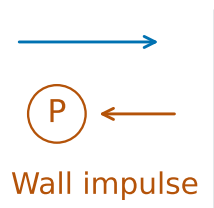
Two-sphere system



Internal impulses cancel.

External horizontal impulse ≈ 0

Sphere alone at a wall



Normal momentum changes.

Tangential component unchanged.

Conservation requires a stated system and a justified momentum component

Draw the line of impact

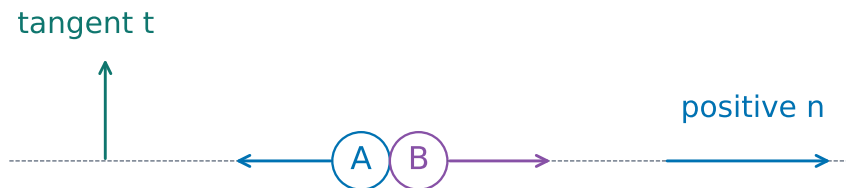
Choose the system and contact direction

For two smooth spheres, the **line of impact** is their **line of centres** at contact. The collision impulse acts along it.

Call that direction normal, n , and a perpendicular direction tangent, t . A **direct impact** has motion along n ; an **oblique impact** also has tangential motion.

Smooth contact changes normal components; each tangential component is unchanged

Smooth contact: impulse acts along the line of centres



Normal: momentum + restitution

Tangent: each component is unchanged

Restitution uses relative normal velocities.

Smooth contact changes normal components; each tangential component is unchanged

Exercises 2.1·2.2

2.1 State the range of possible values of the coefficient of restitution e . [1]

2.2 Two balls approach at a combined closing speed of 8 m s^{-1} with $e = 0.25$. Find their separation speed. [2]

Exercises 2.1·2.2

2.1 State the range of possible values of the coefficient of restitution e . [1]

2.1

- $0 \leq e \leq 1$

2.2 Two balls approach at a combined closing speed of 8 m s^{-1} with $e = 0.25$. Find their separation speed. [2]

2.2

- separation = $e \times$ approach = $0.25 \times 8 = 2 \text{ m s}^{-1}$

Exercise 3.1

3.1 A perfectly elastic collision has a coefficient of restitution:

[1]

☐

A 0

B 0.5

C 1

D 2

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[1]

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A 0

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3.1 C

- A perfectly elastic collision has $e = 1$

Exercise 3.2 (a)

3.2 A sphere P of mass 3 kg moving at 4 m s^{-1} collides directly with a sphere Q of mass 1 kg moving towards it at 2 m s^{-1} . The coefficient of restitution is 0.5.

(a) Write the momentum and restitution equations.

[3]

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(a) Write the momentum and restitution equations. [3]

(a)

- take P 's direction positive: momentum $3(4) + 1(-2) = 3v_P + 1v_Q$, i.e. $3v_P + v_Q = 10$;
restitution $v_Q - v_P = 0.5(4 - (-2)) = 3$

Exercise 3.2 (b)

3.2 A sphere P of mass 3 kg moving at 4 m s^{-1} collides directly with a sphere Q of mass 1 kg moving towards it at 2 m s^{-1} . The coefficient of restitution is 0.5.

(b) Find the velocities of P and Q after impact.

[4]

Exercise 3.2 (b)

3.2 A sphere P of mass 3 kg moving at 4 m s^{-1} collides directly with a sphere Q of mass 1 kg moving towards it at 2 m s^{-1} . The coefficient of restitution is 0.5.

(b) Find the velocities of P and Q after impact. [4]

(b)

- from the two equations: $v_Q = v_P + 3$; $3v_P + v_P + 3 = 10 \Rightarrow 4v_P = 7 \Rightarrow v_P = 1.75 \text{ m s}^{-1}$;
 $v_Q = 4.75 \text{ m s}^{-1}$

Exercise 3.3

3.3 A ball is dropped from a height of 2 m onto the floor, with coefficient of restitution 0.8 between ball and floor. Find the height of the first bounce. [4]

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3.3

- speed hitting floor: $v = \sqrt{2g(2)} = \sqrt{40} = 6.32 \text{ m s}^{-1}$; rebound speed = $ev = 0.8(6.32) = 5.06 \text{ m s}^{-1}$; bounce height = $\frac{(ev)^2}{2g} = \frac{25.6}{20} = 1.28 \text{ m}$