

Exercise walk-through

3.3

Circular motion

A-Level Further Mathematics

A turn needs a change in velocity

The string pulls the ball inwards; the ball pulls your hand outwards. Those forces act on **different objects**.

Choose the ball as the system

Draw the string's force on the ball. Do not put the force on your hand into the ball's free-body diagram.

In a ground-fixed inertial frame, use real interactions. Inertia is not an extra outward force; a rotating-frame centrifugal force is not needed.

Angular speed comes from arc length

An angle θ in **radians** sweeps distance $s = r\theta$. At fixed radius,

$$\text{linear speed} = r \times \text{angular speed}, \quad v = r\omega.$$

Two points on one rotating wheel

They share the same angular speed. A point twice as far from the axis travels twice as far each turn and has twice the linear speed.

Convert degrees to radians before using $v = r\omega$.

Worked: convert a revolution rate

The **period** τ is time per turn; **frequency** f is turns per second.

$$\omega = 2\pi f = \frac{2\pi}{\tau}.$$

30 revolutions per minute; orbit radius 0.4 m

$$f = \frac{30}{60} = \frac{1}{2}, \quad \omega = \pi \text{ rad/s}, \quad v = 0.4\pi \text{ m/s}, \quad \tau = 2 \text{ s}.$$

Revolutions per minute and radians per second are different units.

Inward acceleration is a radial component

Velocity is tangent to the circle. The **centripetal acceleration** is the inward radial component:

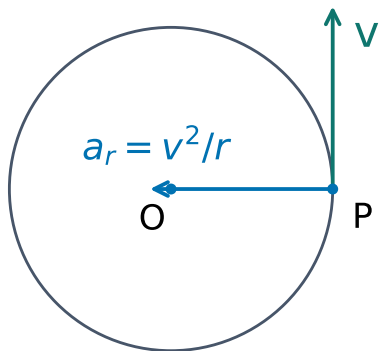
$$a_r = \frac{v^2}{r} = r\omega^2.$$

At constant speed this is the whole acceleration. When speed changes, a **tangential component** is also present.

In a vertical circle under gravity, the full acceleration need not point exactly towards the centre. A proof of the radial formula is not required.

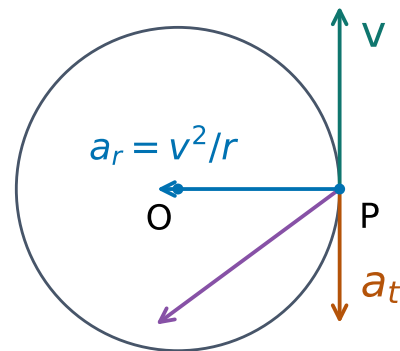
Changing speed adds a tangential component

Constant speed



Total acceleration is radial.

Changing speed



Total a

The radial component changes direction; the tangential component changes speed.

Exercises 2.1·2.2

2.1 A particle moves in a circle of radius 0.5 m at 4 m s^{-1} . Find its acceleration. [2]

2.2 Convert an angular speed of 120 rev/min to rad s^{-1} . [2]

Exercises 2.1·2.2

2.1 A particle moves in a circle of radius 0.5 m at 4 m s^{-1} . Find its acceleration. [2]

2.1

- $$a = \frac{v^2}{r} = \frac{16}{0.5} = 32 \text{ m s}^{-2}$$

2.2 Convert an angular speed of 120 rev/min to rad s^{-1} . [2]

2.2

- $$120 \text{ rev/min} = 2 \text{ rev/s} = 2 \times 2\pi = 4\pi = 12.6 \text{ rad s}^{-1}$$

Exercise 3.1

3.1 The centripetal acceleration of a particle moving at speed v in a circle of radius r is: [1]

☐

A $\frac{v}{r}$

B $\frac{v^2}{r}$

C vr

D $\frac{r}{v^2}$

Exercise 3.1

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A $\frac{v}{r}$

B $\frac{v^2}{r}$

C vr

D $\frac{r}{v^2}$

3.1 B

- $a = \frac{v^2}{r}$ (also $= r\omega^2$)

Exercise 3.2 (a)

3.2 A particle of mass 0.2 kg is attached to a string of length 0.5 m and whirled in a horizontal circle on a smooth table at 3 rev s^{-1} .

(a) Find the angular speed in rad s^{-1} .

[2]

Exercise 3.2 (a)

3.2 A particle of mass 0.2 kg is attached to a string of length 0.5 m and whirled in a horizontal circle on a smooth table at 3 rev s^{-1} .

(a) Find the angular speed in rad s^{-1} .

[2]

(a)

- $\omega = 3 \times 2\pi = 6\pi = 18.8 \text{ rad s}^{-1}$

Exercise 3.2 (b)

3.2 A particle of mass 0.2 kg is attached to a string of length 0.5 m and whirled in a horizontal circle on a smooth table at 3 rev s^{-1} .

(b) Find the tension in the string.

[3]

Exercise 3.2 (b)

3.2 A particle of mass 0.2 kg is attached to a string of length 0.5 m and whirled in a horizontal circle on a smooth table at 3 rev s^{-1} .

(b) Find the tension in the string.

[3]

(b)

- $T = mr\omega^2 = 0.2 \times 0.5 \times (6\pi)^2 = 0.2 \times 0.5 \times 355.3 = 35.5 \text{ N}$

Exercise 3.3 (a)

3.3 A conical pendulum has a bob of mass 0.4 kg on a string of length 1 m, which makes an angle of 30° with the vertical.

(a) Find the tension in the string.

[3]

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(a) Find the tension in the string.

[3]

(a)

- vertical: $T \cos 30^\circ = mg = 4 \Rightarrow T = \frac{4}{0.866} = 4.62 \text{ N}$

Exercise 3.3 (b)

3.3 A conical pendulum has a bob of mass 0.4 kg on a string of length 1 m , which makes an angle of 30° with the vertical.

(b) Find the angular speed of the bob.

[3]

Exercise 3.3 (b)

3.3 A conical pendulum has a bob of mass 0.4 kg on a string of length 1 m, which makes an angle of 30° with the vertical.

(b) Find the angular speed of the bob.

[3]

(b)

- horizontal: $T \sin 30^\circ = mr\omega^2$ with $r = 1 \times \sin 30^\circ = 0.5$; $4.62 \times 0.5 = 0.4 \times 0.5 \times \omega^2 \Rightarrow \omega^2 = 11.55 \Rightarrow \omega = 3.40 \text{ rad s}^{-1}$