

Exercise walk-through

3.2

Equilibrium of a rigid body

A-Level Further Mathematics

Model the flight before calculating

A ball is thrown across a court. Predict its position, speed and landing point.

- Model a **projectile** as a particle moving freely under constant gravity.
- Ignore air resistance. This is less accurate for a light shuttlecock or strong wind.
- Put the origin at launch: rightwards x , upwards y .

Take $g = 10 \text{ m s}^{-2}$ in these examples. State the model's limitations.

Resolve velocity and share one clock

For speed u at angle α above horizontal, $u_x = u \cos \alpha$, $u_y = u \sin \alpha$.

$$x = u_x t,$$

$$y = u_y t - \frac{1}{2} g t^2,$$

$$v_x = u_x,$$

$$v_y = u_y - g t.$$

Worked: $u = 20 \text{ m s}^{-1}$, $\tan \alpha = 4/3$

A 3–4–5 triangle gives $u_x = 12$, $u_y = 16$ in metres per second.

Horizontal acceleration is zero; vertical acceleration is $-g$. Both directions use the same time.

Worked: position is different from velocity

Use $u_x = 12$, $u_y = 16$, $g = 10$ at $t = 1.5$ s.

$$\begin{aligned}x &= 12(1.5) = 18 \text{ m}, & y &= 16(1.5) - 5(1.5)^2 = 12.75 \text{ m}, \\v_x &= 12, & v_y &= 16 - 10(1.5) = 1.\end{aligned}$$

Speed $= \sqrt{12^2 + 1^2} = \sqrt{145} \text{ m s}^{-1}$. Direction $= \tan^{-1}(1/12) = 4.76^\circ$ above horizontal.

Use velocity components for speed and direction; use position components for location.

Worked: a direction fixes signed velocity

The same ball later travels at 45° to the downward vertical.

$$v_y = -v_x = -12,$$

$$t = 2.8 \text{ s},$$

$$16 - 10t = -12,$$

$$(x, y) = (33.6, 5.6) \text{ m}.$$

Its speed is $12\sqrt{2} \text{ m s}^{-1}$. For angle β below horizontal: $v_y = -v_x \tan \beta$. For angle γ to downward vertical: $v_y = -v_x \cot \gamma$.

Read the reference direction and the sign before applying tangent.

Worked: one height can give two times

At height 10 m, the same ball satisfies

$$16t - 5t^2 = 10 \quad \Rightarrow \quad t = \frac{8 \pm \sqrt{14}}{5} \text{ s.}$$

Both times are positive. The corresponding positions are

$$x = \frac{96 \pm 12\sqrt{14}}{5} \text{ m.}$$

The earlier vertical velocity is $+2\sqrt{14}$; the later is $-2\sqrt{14}$. Both speeds equal $\sqrt{144 + 56} = 10\sqrt{2} \text{ m s}^{-1}$.

Keep both roots until “ascending”, “descending” or another event selects one.

Exercises 2.1·3.1

2.1 A force of 20 N acts 3 m from a pivot, at right angles. Find its moment. [2]

2.2 State the two conditions for a rigid body to be in equilibrium. [2]

3.1 The moment of a force about a point is force multiplied by: [1]

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A time

B the perpendicular distance

C mass

D velocity

Exercises 2.1·3.1

2.1 A force of 20 N acts 3 m from a pivot, at right angles. Find its moment. [2]

2.1

- $\text{moment} = 20 \times 3 = 60 \text{ N m}$

2.2 State the two conditions for a rigid body to be in equilibrium. [2]

2.2

- the resultant force is zero; the resultant moment about any point is zero

3.1 The moment of a force about a point is force multiplied by: [1]

☐

A time

B the perpendicular distance

C mass

D velocity

3.1 B

- $\text{Moment} = \text{force} \times \text{perpendicular distance}$

Exercise 3.2 (a)

3.2 A uniform ladder AB of weight 200 N and length 6 m rests with A on rough ground and B against a smooth vertical wall, at 60° to the horizontal.

(a) By taking moments about A , find the reaction at the wall.

[4]

Exercise 3.2 (a)

3.2 A uniform ladder AB of weight 200 N and length 6 m rests with A on rough ground and B against a smooth vertical wall, at 60° to the horizontal.

(a) By taking moments about A , find the reaction at the wall. [4]

(a)

- wall reaction S is horizontal at B ; weight 200 N acts at the midpoint (3 m along).
Moments about A : $S \times 6 \sin 60^\circ = 200 \times 3 \cos 60^\circ$; $S \times 5.196 = 300 \Rightarrow S = 57.7 \text{ N}$

Exercise 3.2 (b)

3.2 A uniform ladder AB of weight 200 N and length 6 m rests with A on rough ground and B against a smooth vertical wall, at 60° to the horizontal.

(b) Hence find the friction force at the ground.

[2]

Exercise 3.2 (b)

3.2 A uniform ladder AB of weight 200 N and length 6 m rests with A on rough ground and B against a smooth vertical wall, at 60° to the horizontal.

(b) Hence find the friction force at the ground.

[2]

(b)

- horizontal equilibrium: friction = $S = 57.7$ N

Exercise 3.3

3.3 A uniform rod AB of length 2 m and weight 40 N rests horizontally on supports at A and at C , where C is 1.5 m from A . Find the reaction at each support. [4]

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3.3 A uniform rod AB of length 2 m and weight 40 N rests horizontally on supports at A and at C , where C is 1.5 m from A . Find the reaction at each support. [4]

3.3

- moments about A : $R_C \times 1.5 = 40 \times 1 \Rightarrow R_C = 26.7 \text{ N}$; vertical: $R_A + R_C = 40 \Rightarrow R_A = 13.3 \text{ N}$