

Exercise walk-through

3.1

Motion of a projectile

A-Level Further Mathematics

A turn needs a change in velocity

The string pulls the ball inwards; the ball pulls your hand outwards. Those forces act on **different objects**.

Choose the ball as the system

Draw the string's force on the ball. Do not put the force on your hand into the ball's free-body diagram.

In a ground-fixed inertial frame, use real interactions. Inertia is not an extra outward force; a rotating-frame centrifugal force is not needed.

Angular speed comes from arc length

An angle θ in **radians** sweeps distance $s = r\theta$. At fixed radius,

$$\text{linear speed} = r \times \text{angular speed}, \quad v = r\omega.$$

Two points on one rotating wheel

They share the same angular speed. A point twice as far from the axis travels twice as far each turn and has twice the linear speed.

Convert degrees to radians before using $v = r\omega$.

Worked: convert a revolution rate

The **period** τ is time per turn; **frequency** f is turns per second.

$$\omega = 2\pi f = \frac{2\pi}{\tau}.$$

30 revolutions per minute; orbit radius 0.4 m

$$f = \frac{30}{60} = \frac{1}{2}, \quad \omega = \pi \text{ rad/s}, \quad v = 0.4\pi \text{ m/s}, \quad \tau = 2 \text{ s}.$$

Revolutions per minute and radians per second are different units.

Inward acceleration is a radial component

Velocity is tangent to the circle. The **centripetal acceleration** is the inward radial component:

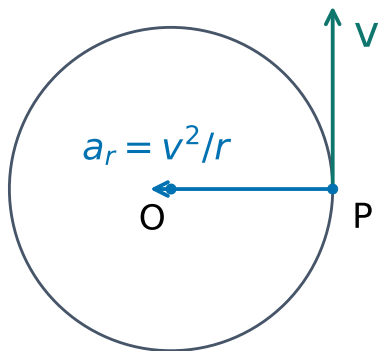
$$a_r = \frac{v^2}{r} = r\omega^2.$$

At constant speed this is the whole acceleration. When speed changes, a **tangential component** is also present.

In a vertical circle under gravity, the full acceleration need not point exactly towards the centre. A proof of the radial formula is not required.

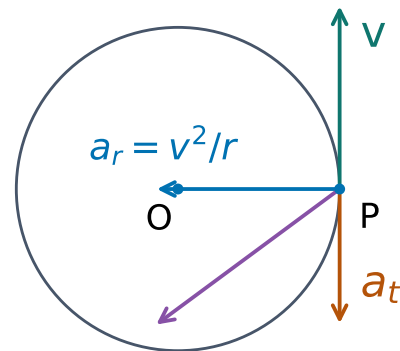
Changing speed adds a tangential component

Constant speed



Total acceleration is radial.

Changing speed



Total a

The radial component changes direction; the tangential component changes speed.

Exercises 2.1·2.2

2.1 A projectile is launched at 25 m s^{-1} at 40° . Find its initial vertical velocity component. [2]

2.2 Write down the formula for the range on level ground. [1]

Exercises 2.1·2.2

2.1 A projectile is launched at 25 m s^{-1} at 40° . Find its initial vertical velocity component. [2]

2.1

- $25 \sin 40^\circ = 25 \times 0.643 = 16.1 \text{ m s}^{-1}$

2.2 Write down the formula for the range on level ground. [1]

2.2

- $\frac{u^2 \sin 2\alpha}{g}$

Exercise 3.1

3.1 A ball is projected horizontally at 15 m s^{-1} from a height. After 2 s its horizontal distance is: [1]

☐

A 15 m

B 30 m

C 20 m

D 7.5 m

Exercise 3.1

3.1 A ball is projected horizontally at 15 m s^{-1} from a height. After 2 s its horizontal distance is: [1]

☐

A 15 m

B 30 m

C 20 m

D 7.5 m

3.1 B

- $x = 15 \times 2 = 30 \text{ m}$

Exercise 3.2 (a)

3.2 A particle is projected at 30 m s^{-1} at 60° to the horizontal.

(a) Find the time to reach the greatest height.

[2]

Exercise 3.2 (a)

3.2 A particle is projected at 30 m s^{-1} at 60° to the horizontal.

(a) Find the time to reach the greatest height. [2]

(a)

- at greatest height the vertical velocity is 0: $0 = 30 \sin 60^\circ - 10t \Rightarrow t = \frac{25.98}{10} = 2.60 \text{ s}$

Exercise 3.2 (b)

3.2 A particle is projected at 30 m s^{-1} at 60° to the horizontal.

(b) Find the greatest height reached.

[3]

Exercise 3.2 (b)

3.2 A particle is projected at 30 m s^{-1} at 60° to the horizontal.

(b) Find the greatest height reached.

[3]

(b)

- $$\text{height} = \frac{u^2 \sin^2 \alpha}{2g} = \frac{900 \times 0.75}{20} = 33.75 \text{ m}$$

Exercise 3.3 (a)

3.3 A stone is thrown at 20 m s^{-1} at 45° from the top of a cliff. It lands in the sea 50 m horizontally away.

(a) Find the time of flight.

[3]

Exercise 3.3 (a)

3.3 A stone is thrown at 20 m s^{-1} at 45° from the top of a cliff. It lands in the sea 50 m horizontally away.

(a) Find the time of flight.

[3]

(a)

- horizontal: $50 = 20 \cos 45^\circ t = 14.14t \Rightarrow t = 3.54 \text{ s}$

Exercise 3.3 (b)

3.3 A stone is thrown at 20 m s^{-1} at 45° from the top of a cliff. It lands in the sea 50 m horizontally away.

(b) Find the height of the cliff above the sea.

[3]

Exercise 3.3 (b)

3.3 A stone is thrown at 20 m s^{-1} at 45° from the top of a cliff. It lands in the sea 50 m horizontally away.

(b) Find the height of the cliff above the sea. [3]

(b)

- take up positive with launch at $y = 0$; at landing $y = -h$: $-h = 20 \sin 45^\circ(3.54) - 5(3.54)^2$; $= 50.0 - 62.5 = -12.5$, so the cliff is $h = 12.5 \text{ m}$ above the sea